STATE CONTROLLER DESIGN OF THE ACTIVELY CONTROLLED DRIVER’S SEAT

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Summary: In the paper the introduction studies of the state controller design of active vibration isolation system of the driver seat are presented. The driver seat with the air spring as the actuator is described as nonlinear lumped system with transportation delay. The linearization of this nonlinear dynamic system is used for discrete linear state space controller design. The behavior of this controller with linear continuous estimator of nonlinear system has been verified in the laboratory.

Key words: driver’s seat, active vibration isolation, state controller, estimator.

INTRODUCTION

Main contribution of the active vibration isolation system – also described in (1), 2 and 3) – is decreasing of vibrations transmissibility for low frequencies and also remaining the low transmissibility for high frequencies. In designed system is not used damper. Contrary of the commonly described systems (4, 5, 6, 7 and 8) in the designed system is used the electronically controlled servo-valve, which feeds the air into the bellows air spring, or discharges the air from the spring into the atmosphere.

We present the nonlinear mathematical model with concentrated parameters of the driver’s seat with an air spring. The linearization of this model is main idea of state space linear controller design. The active vibration isolation is based on feedback principle.

1. MATHEMATICAL MODEL

1.1 Nonlinear model of the driver's seat

Simple mechanical scheme of the considered driver’s seat is shown in fig. 1. Hydraulic damper is not used.

Fig. 1 – Scheme of vibration isolation system with scissor mechanism

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We introduce a following basic notation (see fig. 1): absolute deflection of lower base (kinematic excitation) \( z_1(t) \), absolute deflection of upper base \( z_2(t) \), relative deflection \( z_{2r}(t) \) and reduced mass of upper base (with reduced part of human body) \( M \).

Equation of dynamic forces equilibrium on the system is

\[
\frac{d^2 z_2}{dt^2} = \frac{1}{M} (S_{ef}(p_2 - p_a)) - g - \frac{k_d}{M} \left( \frac{dz_2}{dt} - \frac{dz_1}{dt} \right),
\]

where \( M \) is a driver reduced mass, \( p_2 \) the absolute pressure inside the spring, \( p_a \) the absolute atmosphere pressure, \( g \) the gravity acceleration constant, \( k_d \) the coefficient of viscous friction, \( S_{ef} = h_1(z_2 - z_1) \) the effective area of the air spring and \( h_1 \) the function of distance \( z_2 - z_1 \).

Air mass flow \( Q_m \) filling the air spring

\[
Q_m(t) = u_1(t - \tau_d) k_{v1} \sqrt{p_1[p_1 - p_2(t)]}, \quad u_1 \geq 0,
\]

where \( u_1 \) is the voltage input of electro-magnetic valve (controller output), \( \tau_d \) is transport delay and \( p_1 \) the absolute pressure inside the accumulator.

Air mass flow leaving the air spring into the atmosphere

\[
Q_m(t) = u_1(t - \tau_d) k_{v2} \sqrt{p_2(t)[p_2(t) - p_a]}, \quad u_1 < 0.
\]

\( k_{v1} \) and \( k_{v2} \) are experimentally determined flow coefficients (10).

The time derivative of pressure \( p_2 \) inside the air spring

\[
\frac{dp_2}{dt} = \kappa \frac{Q_m}{m} - \frac{1}{V} \frac{dV}{dt},
\]

where \( \kappa \) is an adiabatic air constant, \( m \) the air mass inside the spring, \( V \) is the spring’s inside volume, \( h_3 \) and \( h_4 \) are the functions of distance \( z_2 - z_1 \)

\[
\frac{dV}{dt} = \frac{dV}{d(z_2 - z_1)} \frac{d(z_2 - z_1)}{dt},
\]

\[
V = h_3(z_2 - z_1), \quad \frac{dV}{d(z_2 - z_1)} = h_4(z_2 - z_1).
\]

We can also write

\[
\frac{dm}{dt} = Q_m.
\]

It is possible to use inside the controller the function, which makes linearization of nonlinear air mass flow (2), (3).

In the fig. 2 are the used variables renamed. With renamed variables the equations (5), (4) are

\[
\frac{dx_1}{dt} = u_1, \quad (6a)
\]

\[
\frac{dx_2}{dt} = \kappa x_2 \left( \frac{u_1}{x_1} - \frac{(x_4 - u_2) h_4(x_3 - x_2)}{h_3(x_3 - x_1)} \right), \quad (6b)
\]

Next equation arises from fig. 2

\[
\frac{dx_3}{dt} = x_4. \quad (6c)
\]
Equation (1) with renamed variables is
\[
\frac{dx_i}{dt} = \frac{1}{M} \left[ x_2 h_1(x_2 - x_3) - p_s h_2(x_1 - x_3) - h_d(x_4 - u_2) \right] - g . \tag{6d}
\]

Last equation of nonlinear model is
\[
\frac{dx_5}{dt} = u_2 . \tag{6e}
\]

In equations (6) are \( x_i, i = 1, \ldots, 5 \), state variables, \( u_1 \) is controller output, \( u_2 \) is disturbance, \( u_2 = \frac{dz_1}{dt} \).

The discussed five equations are nonlinear state equations of the system. The vector form of them is
\[
\dot{x} = f(x, u) . \tag{7}
\]

### 1.2 Linearization

The state equations (7) can be linearized about the operating point \((x_0, u_0)\). The linearization of \(i\)-th state equation is
\[
\dot{\tilde{x}}_i = f_i(x_0, u_0) + \left( \sum_{j=1}^{i-1} \frac{\partial f_i}{\partial x_j} \right) (x_j - x_{j0}) + \left( \sum_{k=0}^{i-1} \frac{\partial f_i}{\partial x_k} \right) (u_k - u_{k0}) . \tag{8}
\]

Let we designate \( \tilde{x}_j = x_j - x_{j0} \), \( \dot{\tilde{x}}_j = \dot{x}_j \) and \( \tilde{u}_k = u_k - u_{k0} \). The linearized state equations are
\[
\dot{\tilde{x}} = A\tilde{x} + B\tilde{u} + f(x_0, u_0) \tag{9}
\]
and the linearized state equations of nonlinear equations (6) are
\[
\begin{align*}
\tilde{x}_1(t) &= \tilde{u}_1(t - \tau_d) + f_1(x_0, u_0) , \\
\tilde{x}_2(t) &= \kappa \left\{ -x_20 \frac{u_{10}}{x_{10}} \tilde{x}_1 + \frac{u_{10}}{x_{10}} \frac{w_2 h_4(w_1)}{h_3(w_1)} \right\} \tilde{x}_2 + x_20 \left[ \frac{w_2 h_4^2(w_1)}{h_3^2(w_1)} - \frac{w_2 h_4(w_1)}{h_3(w_1)} \right] \tilde{x}_3 - \\
&\quad - x_20 \frac{h_4(w_1)}{h_3(w_1)} \tilde{x}_4 - x_20 \left[ \frac{w_2 h_4^2(w_1)}{h_3^2(w_1)} - \frac{w_2 h_4(w_1)}{h_3(w_1)} \right] \tilde{x}_5 + \\
&\quad + \kappa x_20 \left[ \frac{1}{x_{10}} \tilde{u}_1 + \frac{h_4(w_1)}{h_3(w_1)} \tilde{u}_2 \right] + f_2(x_0, u_0) \tag{10a}
\end{align*}
\]
\[ \dot{x}_3(t) = \ddot{x}_3(t) + f_3(x_6, u_0), \]  
\[ \dot{x}_4(t) = \frac{1}{M} \left[ h_1(w_1) \ddot{x}_2 + (x_{20} - p_a) h_2(w_1) \ddot{x}_3 - k_d \ddot{x}_4 - (x_{20} - p_a) h_2(w_1) \ddot{x}_3 + \frac{k_d}{M} \ddot{u}_2 + f_4(x_0, u_0) \right], \]  
\[ \ddot{x}_5(t) = \ddot{u}_2 + f_5(x_0, u_0). \]

where \( w_1 = x_{30} - x_{30} \), \( w_2 = x_{40} - u_{20} \) and 
\[ h_2(w_1) = \frac{dh_1(w_1)}{dw_1}, \quad h_3(w_1) = \frac{dh_2(w_1)}{dw_1}, \quad h_4(w_1) = \frac{dh_3(w_1)}{dw_1}. \]

The linearized state space equations (10) in equilibrium state \( \dot{x} = f(x_0, u_0) = 0 \) were used for linear state space controller design. The modification of this controller was used for control of laboratory driver seat.

2. STATE CONTROLLER

General scheme of the control is shown on fig. 3 and fig. 4. It uses signals from accelerometers 6, 7, displacement sensor 8 and pressure sensor 9 as inputs. Control computer 10 controls the position of the electro-pneumatic valve 3, which governs in- resp. outflow from the air spring 2. Use of any damping device is prohibited, as it would impair transmissibility at higher frequencies. Main problem lies in the use of air for working medium, because of its compressibility.

Fig. 3 – General arrangement of the actively controlled seat

Source: Author, (1)
General task on the seat transmissibility can be therefore posed. It demands achievement of amplitude transmissibility near or somewhat less to one at frequencies 0 to approx. 1 Hz and at least same or better than that of undamped passive 1-DOF at frequencies over approx. 3 Hz. This can be achieved with active control of the seat suspension only.

Fig. 4 – Basic scheme of the control

Detailed scheme of the structure of the controller used in the actively controlled seat is shown on fig. 5. The controller is implemented by a weighted sum of the values \( v_1, a_2, p_2, a_{2E} \) and time integral of the difference \((z_{2r} - z_{2rW})\), \(z_{2rW}\) is the required distance (setpoint value) from the seat base. Quantities \( a_2, p_2 \) and \( z_{2r} \) are measured directly on the seat. Seat base speed \( v_1 \) is calculated from the measured values \( a_1 \), which is at first filtered (high-frequency filter with transmission \( G_f(s) \)) and then is integrated in time. Value \( a_{2E} \) is computed by the relative motion estimator. \( k_{e1} \) and \( k_{e2} \) are estimator coefficients, \( G_{e1}(s) \) and \( G_{e2}(s) \) are transmissions of high-frequency filters.

Fig. 5 – Detailed scheme of the state controller implementation
The state feedback control law can be written as

$$u(t) = -k_i \left[ z_{2i}((\tau) - z_{2i}w) \right] d\tau - k_2 p_2(t) - k_3 a_2(t) - k_4 v_1(t) - k_5 a_2(t) ,$$

(11)

where $k_i$ (for $i = 1, \ldots, 5$) are state controller coefficients, $u(t)$ is controller output. The resulting voltage generated by the controller is calculated

$$U_r = u_t + U_0 ,$$

(12)

where $u_t$ is the action variable of the nonlinear controller and $U_0 = 2.912$ V is the voltage at which the guaranteed zero air flow through the valve. The resulting control valve voltage

$$U_v = f_k(U_r, p_2, p_t)$$

(13)
The compensation function $f_k$ (see figs. 4 and 5) is referred to fig. 6. It was designed by numerical optimization using dynamic programming methods – discrete version of the Bellman principle of optimality. The built-in compensation function is composed of sections of the second degree polynomials (boundary between marked points) respectively first degree polynomials (outside the boundary points).

The another controller (see fig. 7) is implemented by a weighted sum of the values $a_1, v_1, z_1, a_2, p_2, a_{2rE}, v_{2rE}$ and time integral of the difference ($z_{2r} - z_{2rW}$). Thus, the state feedback control law can be written as

$$u(t) = -k_i \int_0^t \left[ (z_{2r}(\tau) - z_{2rW}) d\tau - k_2 p(t) - k_3 a_2(t) - k_4 v_1(t) - k_5 z_1(t) - k_6 a_{2rE}(t) - k_7 v_{2rE}(t) \right],$$

where $k_i$ (for $i = 1, \ldots, 7$) are state controller coefficients.

Controller action variable $u_t$ is a nonlinear function

$$u_t = f_n(u),$$

Because $U_t = u_t + U_0$, the resulting control valve voltage

$$U_v = f_k(U_t, p_2, p_1).$$
Simulation model of the driver’s seat with nonlinear controller which includes a chain of integrators with high-frequency filters to calculate the velocity and position of the seat base and includes the above described estimator of relative quantities is shown on fig. 8. The simulation model was created in MATLAB – Simulink.

Adjustable state controller coefficients were found by numerical simulation on the mathematical model (see chapter 1 and fig. 7). Controller parameters have been optimized using the simulation model in figure 8.

3. LABORATORY RESULTS

The results of laboratory verification with disturbances measured on truck TATRA 815, during the drive on off-road track, are in fig. 10. For comparison are in fig. 11 presented the measurements with industry produced driver seat with passive vibration isolation system. The used disturbances are in both figures the same.

![Actively controlled seat with 2-DOF dummy on electro-hydraulic test rig](image1)

Source: Author, (1)

![Laboratory measurement with actively controlled seat](image2)

Source: Author

Fig. 9 – Actively controlled seat with 2-DOF dummy on electro-hydraulic test rig

Fig. 10 – Laboratory measurement with actively controlled seat
Fig. 11 – Laboratory measurement with industry produced driver seat

Fig. 12 – PSD of the actively controlled seat base acceleration $a_1$ and seat cushion acceleration $a_2$

Fig. 13 – Acceleration amplitude transmissibility of the actively controlled seat
Seats were loaded by simple weights or by a 2-DOF dummy. Test results were however for simple weights and dummies very similar and their minor differences are not discussed here. Following results were gained with the dummy (see fig. 9). $P_{a_1}$ is the power spectral density (PSD) of the seat base acceleration $a_1$ and $P_{a_2}$ is the PSD of the seat cushion acceleration $a_2$.

**CONCLUSIONS**

Experience gained during the laboratory testing leads to the optimistic conclusion, that relatively big changes in seat transmissibility can be achieved by suitable seat suspension control even on air-sprung seats. The different penalty functions for controller design and structures of the estimators, which are the parts of state space controller, are tested at present. The linearization of system state space equations will be used for nonlinear state space controller design in future. Because of the flexibility of the control system, transmissibility of the actively suspended seat can be tuned to the actual demands of its passengers, if these demands will be known.

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