

# THE OPTIMALIZATION OF LOCATION TRANSPORT JUNCTIONS

## OPTIMALIZACE UMÍSTĚNÍ DOPRAVNÍCH UZLŮ

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*Summary: This article is focused on the issue of deployment of transport junctions and other transport network, using modern optimization algorithms. The main emphasis is placed on genetic algorithm operating on the principle of location with simple hub allocation without capacity constraints and simulation of evolutionary processes applied to the transport nodes.*

*Key words: algorithm, hub, location, material flow optimization*

*Anotace: Tento článek je zaměřen na problematiku lokace dopravních uzlů na dopravní síti, s využitím moderních optimalizačních algoritmů. Hlavní důraz je kladen na využití genetických algoritmů, které pracují na principu umístění s jednoduchým hubem bez omezení kapacity a simulaci evolučních procesů použitých na dopravních uzlech.*

*Klíčová slova: lokace, hub, optimalizace materiálového toku, algoritmus*

### 1. INTRODUCTION

The distribution system can be briefly defined as a transport system that transports goods from sources to customers. In some systems, you can associate individual shipments up as many primary sources and dispatched a relatively small number of items. Subsequently, in larger doses to carry a terminal near the destination, there is a sort, and transported to its destination. This organization is called a transportation hub and spoke. Instead of direct shipments from source to target junctions are used terminals, called hubs. The advantage of hub and spoke organization lies in the concentration of traffic flows into the hub, which allows the hub to transport mail and economically acceptable limits. When processing elements in the hubs can extend the route and there are also extending the delivery periods compared to direct transport, it is appropriate to organize the priority of transportation such as direct transportation. Problem of optimizing the individual packages with higher speed and quality of transport, in conjunction with the search for solutions to economic situation is becoming extremely topical. With the increasing use of the services hub are increasing capacity requirements of existing systems of general cargo transportation. Particularly in

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some regional depots are already at their capacity limit and in terms of further development is necessary to seek a solution to this situation.

## 2. GENETIC ALGORITHM

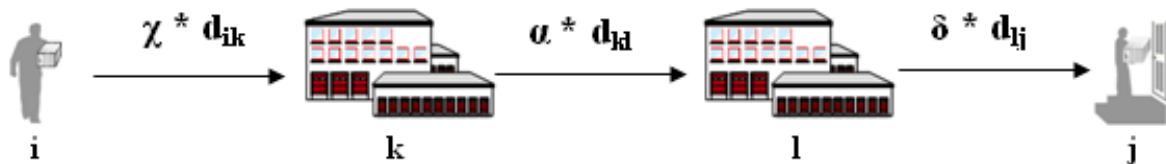
Genetic algorithm is a heuristic method of dealing with applying the simulation of evolutionary processes to generate suboptimal solutions location problems. Group of genetic algorithms used biological processes, such as inheritance, mutation, natural selection and crossbreeding, which are partial operations. At the beginning is population where each individual represents a unique solution to the problem. After the selection of stochastic different individuals through various processes are crossing and mutation, transformed to a new individual and a new population. This process is then repeated iteratively to find than the required suboptimum.

### 2.1 Specification of the problem, the verbal model

Transport network is modelled by a complete graph  $G$  with a set of nodes  $V$  and a set of edges of  $H$ . Each edge is evaluated  $d_{ij}$  number that represents the distance from node  $i$  node  $j$  in the real traffic network. The size of the transport stream from node  $i$  to node  $j$  is labelled as  $b_{ij}$ .

Each shipment between node  $i$  and node  $j$  consists of three components: the movement from node  $i$  to the hub (take-away section), the transport hub for the distribution hub  $l$  and later shipments of up to node  $l$  (take away part).

Transport costs per unit of quantity of flow from node  $i$  to node  $j$  via hubs sludge will be calculated according to the relation  $c_{ij} = \chi * d_{ik} + \alpha * d_{kl} + \delta * d_{lj}$ .



Source: Authors

Fig. 1 - Schematic of unit transport costs

Parameters  $\chi$ ,  $\alpha$ ,  $\delta$  are resolution allowing the costs of collection, transportation and distribution of the Hubs. The parameters  $\chi$  and  $\delta$  are usually equal to 1 (in some applications to distinguish the costs of collection and distribution), the choice of the parameter  $\alpha$  can reflect the amount of savings in transport costs resulting from the concentration of transport between hubs (the value of the parameter  $\alpha$  in the practical problems usually ranges between 0, 6 to 0.7). Transport costs per unit of quantity  $c_{ij}$  may be using the appropriate values of parameters  $\chi$ ,  $\alpha$ ,  $\delta$  are expressed in monetary units with the assumption of linear growth in cost depending on the kilometre distance.

Quantity transmission work done in the transport of quantities  $b_{ij}$  from node  $i$  to node  $j$  via hubs sludge is then calculated as  $b_{ij} * c_{ij}$ .

The aim of the role is to decide on the location of hubs and the assignment (allocation) of these nodes operated hubs so that the total size of the transmission work done was minimal. If resolver knows the fixed costs of building it is possible to formulate a task, which minimizes the total cost, while the number of hubs in the objective function acts as a variable.

## 2.2 Mathematical formulation of the problem

Own decision as to whether the node is assigned to a hub or not, they model the variables  $h_{ik}$ .  $H_{ik}$  value = 1 means that node  $i$  is assigned to the hub, otherwise the value  $h_{ik} = 0$  because each node  $k$ , which becomes Hub, must be assigned to itself, variable  $h_{kk} = 1$  also indicates that the node is the Hub.

*Role with a fixed number of hubs*

Number of hubs is given in advance, identified as  $p$

Min

$$\sum_i \sum_j b_{ij} \left( \sum_k \chi d_{ik} h_{ik} + \sum_k \sum_l \alpha d_{kl} h_{ik} h_{jl} + \sum_l \delta d_{jl} h_{jl} \right) + \sum_k h_{kk} f_k \quad (1)$$

Conditions:

$$\sum_k h_{kk} = p \quad (2)$$

$$\sum_k h_{ik} = 1 \quad \text{pro } i \in V \quad (3)$$

$$h_{ik} \leq h_{kk} \quad \text{pro } i, k \in V \quad (4)$$

$$h_{ik} \in \{0, 1\} \quad \text{pro } i, k \in V \quad (5)$$

Objective function (1) expresses the total cost (sum of transportation work, expressed in monetary units and fixed costs). Condition (2) ensures that exactly  $p$  hubs chosen, condition (3) guarantees that every node will be assigned to a single hub. Condition (4) ensures that all goods are transported only through the nodes, which are established hubs.

In the case of the unknown or the equivalent of fixed costs (such as the newly built system) can be a folder containing the objective function fixed costs omitted. This will not benefit from junctions that already contain the necessary infrastructure land is cheaper in them, etc.

In English literature, this type of problems described as USApHMP (Uncapacitated Single Allocation Hub Median Problem). It is an NP-hard task, its solution is known polynomial algorithm complexity, it is therefore necessary to use any of the heuristic algorithm.

*The role of variable number of hubs*

Number hub acts as variable, fixed costs are always known. Formulation task will look as above with the only difference being that a set of restrictive conditions disappear condition (2).

In the English literature of this type of problem is called USAHLP (Uncapacitated Single Allocation Hub Location Problem). And here is a NP-hard task.

### 2.3 Options for solving the formulated problem

The problem of locating hub with no free allocation of capacity constraints as first formulated by O'Kelly NP-hard problem of quadratic integer programming with non-convex utility functions (1). Has since been developed many heuristic methods based on the principle of Branch-and-Bound, neural networks, Tabu search, simulated annealing and finally genetic algorithms.

### 2.4 Description of the genetic algorithm

That algorithm is mainly based on the algorithm GAHUB2, whose author is J. Kratica and team; it expands the use of a job with a variable number of hubs.

### 2.5 The basic scheme of genetic algorithm

Genetic algorithm is used to find suboptimal solutions to complex combinatorial problems. Operate on the simulation of evolution in nature, which consists of:

- the role of encryption solutions in the form of so-called chromosome (building blocks of chromosomes, genes) and assign a fitness value (a measure of quality of solution, i.e. the value of objective function) to each chromosome,
- creating an initial population (i.e., sets of chromosomes),
- selecting individuals for reproduction (based on their fitness values),
- the process of reproduction (through breeding and mutation operations),
- creation of a new generation.

Repeat the process of simulated evolution until the desired objective function value or a predefined number of generations.

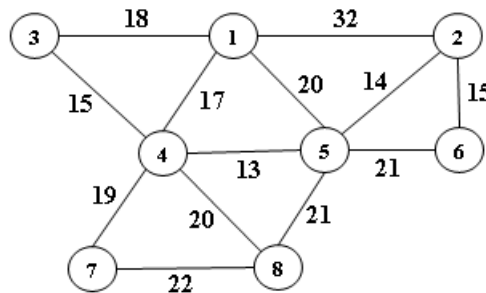
### 2.6 Encoding task

Encode the solution of the shape of the chromosome may be described as follows: length of the chromosome (i.e. the number of genes) is equal to the number of nodes in a graph  $G$ . Each chromosome (corresponding to a single node) consists of two segments. The first segment indicates whether the node or not the hub (i.e. the value of variable  $h_{kk}$ ), the second segment expresses the node assignment to a particular hub. Generally, the assignment to the nearest hub node may not always be optimal (depending on the distribution of traffic flows in the network). For each node is therefore a list of hubs arranged in ascending order of distance from the hub node. If the second segment of the gene takes value 0, the node is assigned to it the next hub, where takes the values 1, is assigned to the second closest hub, etc.

The advantage of the method used to express the coding node allocation for up to preserve the admissibility of the solution even after the crossing and mutation operators (i.e., elimination of invalid links to nodes in the newly acquired solutions are not the mouth).

Example: chromosome 00 | 00 | 00 | 10 | 10 | 00 | 00 | 01 corresponds to the solution in which they are placed in the top hubs 4 and 5 peaks 1, 3, 4 and 7 are assigned to hub 4,

peaks 2, 5, 6 and 8 are assigned to the fifth Corresponding network model is shown in Figure 2.



Source: Authors

Fig. 2 - Model network for the example

## 2.7 Initial population generation

When creating initial population of the desired size (default is set to 150 individuals, you can choose a different size) is used the following strategies.

*For the task with a fixed number of hubs:*

The first segment of each gene takes the value 1 with probability  $p / n$ , where  $p$  is the number of hubs and  $n$  is the total number of nodes in a graph  $G$ . The first generation of the values of segments of genes can happen that the actual number of hubs will differ from the number  $p$ , in that case is the number of hubs in the chromosome repair (repair chain direction is chosen randomly with a probability of 0.5).

The second segment of the gene is preserved in the form of binary numbers, and its default value is all zeros (length of a binary number is a specific instance of the intended role of the required number of hubs). Generating value of second segment of the gene is constructed so that the last bit takes the value 1 with probability  $1 / n$ , each bit further towards the beginning of a binary number takes the value 1 with probability half the size (i.e.,  $1/2n$ ,  $1/4n$  ...). The reason is simple: change the last bit position of the binary number causes change in value of the corresponding decimal number by  $\pm 1$  change in the penultimate position of  $\pm 2$ , change to another position  $\pm 4$ , etc. The strategy should ensure that the vast majority of nodes will assigned to its nearest hub, a smaller number of nodes to the second nearest hub, still less the third node to the nearest hub, etc. This strategy is used for the hubs, which are always assigned to them.

Algorithm allows to create an initial population to take into account the overall size of the transport flow in the nodes - in this case is a list of nodes arranged in descending order of size of the total transport flow in each node. First 75% of the population is created so that the hubs selected only the first two thirds of this list for the remaining 25% are the hubs of the entire set of selected nodes.

*For the task with a variable number of hubs:*

Number of hubs (corresponding to the number  $p$ ) is for 75% of the population is generated in the range of  $1 - n / 4$  for the remaining 25% of the population in the range  $n / 4 - n / 2$ , where  $n$  is the total number of nodes. This strategy is used in (6). Having thus obtained the number of hubs are individual solutions generated, as in the case of tasks with a fixed number of hubs.

## **2.8 The selection process**

Selection, thus selecting individuals for reproduction, always precedes the calculation of objective function - the fitness value for each solution of the task. When creating a new generation of so-called elitism is applied - this means that a defined number (default is 100) the best solution is transferred directly into the next generation. Sense of elitism is to maintain good genetic material.

Selection of individuals who will participate in simulated reproduction is carried out tournament means - i.e., all current generation is selected by a number of individuals (called the size of the tournament, you can choose), "won" the tournament becomes an individual with the lowest fitness. With tournament selection two individuals are drawn on which will be carried out crossing and mutation. The number of these pairs is determined as the difference between the population size and number of elite individuals enrolled directly into the next generation.

## **2.9 The process of reproduction**

*Cross*

Using the operator is generated by crossbreeding combinations of genetic material of two (quality) of individuals with the hope that this combination will be obtained even better solution.

*For the task with a fixed number of hubs:*

Crossing two individuals such that the program goes through the genetic code of both crossbred individuals in the direction from right to left, while looking for a position as to who has the first individual in the first segment of the gene value of 1 and the other individual value of 0. If you find such a position, replace entire genes of both individuals in this position. In parallel with the above-described process, the program goes through the genetic code of both individuals in the opposite direction (from left to right) while looking for a position of  $j$ , which has the first individual in the first segment of the gene value of 0 and the other individual value of 1. If you find such a position, once the genes of individuals exchanged. Both processes occur simultaneously until the moment when  $j \geq i$ . By using the operator described in a cross shape is a guarantee that both the newly established individuals will contain exactly  $p$  hubs. Default probability of crossing the PC is set to 0.85 (i.e. the probability that a new generation of progressing individuals emerging from the crossing, with a probability of 0.15 proceed to a new generation of both the original specimens).

Example:  $p = 3$  (the position at which a change has occurred is marked in bold):

parent1 10 | 00 | 00 | 10 | 10 | 00 | 00 | 01 descendant1 10 | **10** | **10** | **01** | 00 | 00 | 00 | 01  
 parent2 01 | 10 | 10 | 01 | 00 | 10 | 01 | 00 descendant2 01 | **00** | **00** | **10** | **10** | 10 | 01 | 00

*For the task with a variable number of hubs:*

In this case, the intersection is so that the randomly generated number  $r$  from 1 to  $n-1$  where  $n$  is the number of nodes. Thus generated the position  $r$  "crosses." two chromosomes. This procedure does not warrant maintaining the same number of hubs, which of course it is not desirable.

Example:  $r = 4$  (the position at which a change has occurred is marked in bold):

parent1 10 | 00 | 00 | 01 | 10 | 00 | 00 | 01 descendant 1 10 | 00 | 00 | 01 | **00** | **10** | **01** | **00**  
 parent2 01 | 10 | 10 | 01 | 00 | 10 | 01 | 00 descendant 2 01 | 10 | 10 | 01 | **10** | **00** | **00** | **01**

## 2.10 Mutation

The purpose of the mutation operator is to produce yet unexplored or lost genetic material and thus preventing premature convergence to local optimum role. Both segments of each gene from the ground with a (very small) probability altered. Probability of mutation for the first segment of each gene is  $PM1 / n$ , where  $n$  is the number of nodes. Mutation probability for the second segment of each gene is  $PM2 / n$  for the last bit, for every bit of forward mutation probability is two times lower (i.e.  $0.05 / n$ ,  $0.025 / n$  ...). Based on previous numerical tests in the algorithm uses the values GAHUB2  $PM1 = PM2 = 0.4$  and  $0.1$ .

Mutation operator can use when solving problems with fixed number of hubs cause the number of hubs will be different from the number of  $P$ . Therefore, each chromosome checked and if necessary, add / remove the appropriate number of hubs.

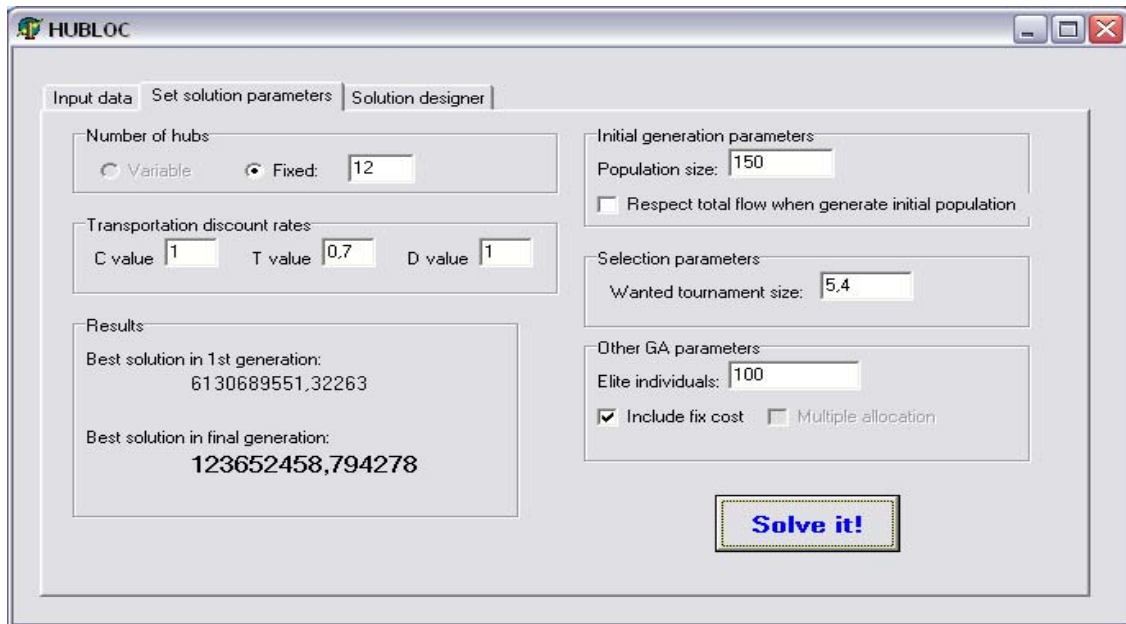
During the genetic algorithm may happen that genes are a large majority of subjects in some of the same position. Such genes are called "frozen" and their presence is undesirable, because the role it can prematurely converge to some local optimum. Therefore, the algorithm is supplemented by monitoring the occurrence of these "frozen" genes. When they found for these genes significantly increased the probability of mutation.

Number of generations can be arbitrarily selected, the default value is 500, which is sufficient for smaller instances task (to about 60 knots), for instance greater role should be to increase the number of generations.

## 2.11 Program HubLoc

The genetic algorithm was working as a specially created program HubLoc, which was compiled at the Department of Technology and Control, University of Pardubice.





Source: Authors

Fig. 3 - HubLoc – input parameters of genetic algorithm

Program takes input data from a text file, which is based on file format CSV (semicolon delimited data). After running the algorithm the program displays the value of the objective function corresponding to that found the best solution. Detailed results, including solution design program is exported to a CSV file, which can be viewed as in MS Excel.

Apart from the optimization program allows you to design custom solutions and calculate the corresponding value of the objective function.

### 3. CONSLUSIONS

Optimization of transportation hubs in the hub and spoke system is very timely in the context of growing demands for quality and the speed of transport. The current trend of concentrating traffic flows into larger and economically less demanding traffic sessions entails the problem of distribution hubs. There are many possible algorithms. Majority, however, these algorithms are heuristic algorithms. This post has provided a detailed description of the procedure, genetic algorithm, which is the appropriate response to address the optimization of transportation hubs.

### 4. ACKNOWLEDGEMENT

This paper has been supported by the Institutional research “Theory of transport systems” (MSM 0021627505) on Jan Perner Transport Faculty, University of Pardubice, Czech Republic and the project “Optimization of collection and delivery of small packages by road and rail transport” (CG932-019-520) on Ministry of Transport of the Czech Republic.



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