MODELS OF AIR TRAFFIC - REGRESSION MODELS

Rudolf Volner[1], Daša Tichá[2]

Summary: Traffic models are at the heart of any performance evaluation of information data (information data about stream aircraft, about stream passengers in airport terminal, information data about luggage, etc.). An accurate estimation of network performance is critical for the success of data networks. Such networks need to guarantee an acceptable quality of service (QoS) level to the users. Therefore, traffic models need to be accurate and able to capture the statistical characteristics of the actual traffic.
Key words: Air traffic, regression, model

1. INTRODUCTION

Performance modeling techniques are needed to determine which congestion control techniques should be used. Performance modeling techniques include:

- analytical techniques,
- computer simulation,
- experimentation.

Performance models require accurate traffic models which can capture the statistical charakteristics of actual traffic. If the traffic models do not accurately represent actual traffic, one may overestimate or underestimate network performance.

2. REGRESSION MODELS

Regression models define explicitly the next random variable in the sequence by previous ones within a specified time window and a moving average of a deformation. In this section several regression models are presented. These models is possible employ near worked information data about stream aircraft, about stream passengers in airport terminal, information data about luggage, etc.

Autoregressive models

The autoregressive model of order *p*, denoted as $AR(p)$, has the following form

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots.. + \phi_p X_{t-p} + \varepsilon_t \qquad (1)$$

where ε_t - deformation,

ϕ_j - real numbers,

[1] doc. Ing. Rudolf Volner, Csc., VŠB -Technical university of Ostrava, Faculty of Mechanical Engineering, Institute of Transport, Departement of Air Transport, 17. listopadu 15, 708 33 Ostrava - Poruba, Tel.: +420 596 99 17 65, E-mail: rudolf.volner@vsb.cz,
[2] doc. Ing. Daša Tichá, PhD., University of Žilina, Faculty of Electrical Engineering, Department of Telecommunications and Multimedia, Univerzitná 8215/1, 010 26 Zilina, Slovak Republic, Tel./Fax: +421 513 2224, E-mail: dasa.ticha@fel.uniza.sk

X_t - prescribed correlated random variables.

If ε_t is a white Gaussian deformation with variance $\sigma^2_{\varepsilon_t}$, then X_t is will be normally distributed random variables. Let as define a lag operator *B* as $X_{t-1} = BX_t$, and let $\phi(B)$ be a polynomial in the operator *B*, defined as follows

$$\phi(B) = (1 - \phi_1 B - .... - \phi_p B^p)$$

Then, the $AR(p)$ process can be represented as

$$\phi(B)X_t = \varepsilon_t \tag{2}$$

The process $\{X_t\}$ is stationary if the roots of $\phi(B)$ lie outside the unit circle [8]. The auto-correlation ρ_k can be computed by multiplying eq.(1) with X_{t-k}, taking the expectation, and dividing by the variance γ_0

$$\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} + ... + \phi_p \rho_{k-p}\ , \quad for\ k \rangle 0 \tag{3}$$

Thus, the general solution is

$$\rho_k = A_1 G_1^k + A_2 G_2^k + ... + A_p G_p^k \tag{4}$$

Where G_i^{-1} are the roots of $\phi(B)$. Therefore, the auto-correlation function of $AR(p)$ process will consist, in general, of damped exponentials, and/or damped sine waves depending on whether the roots are or imaginary.

Since successive information frames do not vary much visually, AR models have been used to model the output information rate of data encoder. In [2], a information source is approximated by a continuous fluid flow model. In the model, the output information rate within a frame period is constant and changes from frame to frame according to the following AR(1) model

$$\lambda[n] = \phi\lambda[n-1] + b\varepsilon[n] \tag{5}$$

Where $\lambda[n]$ is the information rate during frame *n* and $\varepsilon[n]$ is chosen such that the probability of $\lambda[n]$ being negative is very small. Since the number of bits in frame *n* cannot be negative, the value of $\lambda[n]$ in eq.(5) is set to zero, whenever $\lambda[n]$ is negative. This model, cannot capture abrupt changes in the frame information rates that occur due to scene changes or visual discontinuities. Therefore, one may model the information rate of frames within the scene as an AR process and model the scene changes by an underlying Markov chain.

In [2], information traffic is modeled as

$$X_n = Y_n + Z_n + V_n C_n$$

Y_n and Z_n are two independent AR(1) processes. These two AR processes are used to get a better fit of the auto-correlation function of the empirical data than using only one AR(1) process. The last term, $V_n C_n$ is the product of a 3-rate Markov chain and an independent normal random variable. It is designed to capture sample path spikes due to information data changes. This may work for encoding techniques in which only the difference between the frames and a reference frame is encoded and transmitted. Reference frames are transmitted when the difference is greater than a certain threshold. They, in general, indicate a data change. Reference frames have higher bit rates than the other surrounding frames and they cause the spikes in the sample path of the bit rate of the information stream.

Although it is easy to estimate the AR model parameters and to generate the data stream recursively, the exponential decay of the auto-correlation functions makes the model unable to capture auto-correlation functions that decay at a slower rate than the exponential. AR is approximated in [2] by a Markov modulated fluid model, in order to obtain analytical queuing performance results. AR processes with Gaussian distribution cannot capture information traffic probability distribution.

3. DISCRETE AUTOREGRESSIVE MODELS

A discrete autoregressive model of order *p*, denoted as DAR(*p*), generates a stationary data stream of discrete random variables with an arbitrary probability distribution and with an auto-correlation structure similar to that of an AR(*p*).

DAR(1) is a special case of DAR(*p*) process and it is defined as follows: let $\{V_n\}$ and $\{Y_n\}$ be two data streams of independent random variables. The random variable V_n can take two values 0 and 1, with probabilities, $1-\rho$ and ρ, respectively. The random variable Y_n has a discrete state space *S* and $P\{Y_n = i\} = \pi(i)$. The data stream of random variables $\{X_n\}$ which is formed according to the linear model

$$X_n = V_n X_{n-1} + (1 - V_n)Y_n$$

is a DAR(1) process. DAR(1) process is a Markov chain with discrete state space *S* and a transition matrix

$$P = \rho I + (1-\rho)Q$$

where I – the identity matrix,

Q – matrix with for $Q_{ij} = \pi(j)$ for $i, j \in S$.

DAR(1) has a correlation structure of a first-order autoregressive process with $\rho_k = \rho^k$ and has the probability distribution function of π. In [1], the number of cells per frame of information data is modeled by DAR(1) with negative binomial distribution. The rows of the Q matrix consist of the negative-binomial probabilities $(f_0, f_1, \ldots\ldots, f_k, F_K^c)$, where $F_K^c = \sum_{k \rangle K} f_k$, and *K* is the peak rate. Therefore, only mean, variance, peak rate, and first auto-correlation coefficient are needed to be estimated from the data. DAR(1) has far less number of parameters than the general Markov chains. The parameter estimation is simple. The distribution of the resulting process is arbitrary. Moreover, the analytical queuing performance is tractable. On the other hand, the auto-correlation function decays exponentially and hence it cannot be used to model traffic with a slower auto-correlation decay.

Autoregressive moving average models

An autoregressive moving average model of order (p,q), denoted as ARMA(p,q), has the form

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + \varepsilon_t - \Theta_1 \varepsilon_{t-1} - \Theta_2 \varepsilon_{t-2} - \ldots - \Theta_q \varepsilon_{t-q} \tag{6}$$

which can be equivalently represented as

$$\phi(B) X_t = \Theta(B) \varepsilon_t \tag{7}$$

where B and $\phi(B)$ are as defined previously, and

$$\phi(B) = \left(1 - \Theta_1 B - \ldots\ldots - \Theta_q B^q\right)$$

This is equivalent to filtering a deformation process ε_t by a causal linear shift time invariant filter having a rational system function with *p* poles and q zeros [8], that is

$$H(z) = \frac{B_q(z)}{A_p(z)} = \frac{1 - \sum_{k=0}^{q} \Theta_k z^{-k}}{1 - \sum_{k=1}^{p} \phi_p z^{-k}} \tag{8}$$

The auto/covariance γ_k of the ARMA(*p*,*q*) process can be obtained by multiplying eq.(6) with X_{t-k}, taking the expectation and finding the cross/correlation between ε_t and X_t

$$\gamma_k = \phi_1 \gamma_{k-1} + \ldots. + \phi_p \gamma_{k-p} - \sigma_\varepsilon^2 \left(\Theta_k h_0 + \Theta_{k+1} h_1 + \ldots + \Theta_q h_{q-k}\right) \tag{9}$$

where h_t is the impulse response of the ARMA(*p*,*q*) filter $H(z)$.

Note that $\Theta_k = 0$ for $k \rangle q$, therefore, the auto-correlation of the process for $k \rangle q$

$$\rho = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} + \ldots. + \phi_p \rho_{k-p} \quad \text{for} \quad k \rangle q \tag{10}$$

which is the same difference equation as eq.(3), therefore, the auto-correlation of the ARMA(*p*,*q*) decays exponentially.

The duration of a information stream is equally divided into *m* time intervals. The number of cells in the n^{th} time interval is modeled by the following ARMA process

$$X_n = \phi X_{n-m} + \sum_{k=0}^{m-1} \Theta_k \varepsilon_{n-k}$$

Since information data will correlate at each frame due to temporal correlation, the auto-correlation function has peaks at all lags which are integer multiples of *m*. In the above model, the AR part is used to model the re-correlation effect and the Θ_k's are used to fit the correlation at other lags.

Autoregressive integrated moving average models

The autoregressive integrated moving average model of order (p,d,q), denoted as ARIMA(p,d,q), is an extension to the ARMA(*p*,*q*). It is obtained by allowing the polynomial $\phi(B)$ to have *d* roots equal to unity. The rest of the roots lie outside the unit circle. The ARIMA(p,d,q) has the form

$$\varphi(B) = \nabla^d X_t = \Theta(B) \varepsilon_t \tag{11}$$

where ∇ is a difference operator, defined as

$$(X_t - X_{t-1}) = \nabla X_t$$

and $\varphi(B)$ is a polynomial in *B*.

Notice that

$$\nabla X_t = (1-B)X_t$$

That ARIMA(p,d,q) is used to model homogeneous non-stationary time series. For example, a time series that exhibits non-stationary in level, or in level and slope, can be modeled by using ARIMA$(p,1,q)$ and ARIMA$(p,2,q)$.

4. TES MODELS

Transform-expand-sample (TES) models are non-linear regression models with modulo-1 arithmetic. They aim to capture both auto-correlation and marginal distribution of empirical data. TES models consist of two major TES processes [1, 2]:

- TES^+,
- TES^-.

TES^+ produces a sequence which has positive correlation at lag 1, while TES^- produces a negative correlation at lag 1.

Before describing TES^+ or TES^-, we need to introduce a few definitions and annotations. The modulo-1 of a real number *x*, denoted as $\langle x \rangle$, is defined as

$$\langle x \rangle = x - \lfloor x \rfloor$$

where $\lfloor x \rfloor$ is the maximum integer less than *x*.

Therefore, $\langle x \rangle$ is always non-negative. If the interval [0,1) is viewed as a circle that is obtained by joining the points 0 and 1, one can define a circular interval $C[a,b)$, where a and $b \in [0,1)$, as all the points on the circular unit interval going clockwise from point *a* to point *b*. Therefore,

$$C[a,b) = \begin{cases} [a,b), & if \quad a \le b \\ [0,1) - [b,a), & if \quad a \rangle b \end{cases}$$

TES^+ **and** TES^- - $TES^+(L,R)$ is introduced in [2] and is characterized by two parameters, L and R. The sequence $\{U_n^+\}$ is generated recursively as follows : initialize $U_0^+ = U_0$, where U_0 is uniform in the interval (0,1), Then U_n^+ is uniformly sampled random variable on the circular interval $C_{U_n^+} = [\langle U_{n-1}^+ + L \rangle, \langle U_{n-1}^+ + R \rangle)$.

In the $TES^-(L,R)$, the sequence is generated as in TES^+ with U_n^- is uniform random variable over the circular interval

$$C_{U_{n-1}^-}[a,b) = \begin{cases} [\langle 1 - U_{n-1}^- - L \rangle, \langle 1 - U_{n-1}^- + R \rangle), & n\, even \\ [\langle 1 - U_{n-1}^- - R \rangle, \langle 1 - U_{n-1}^- + L \rangle), & n\ odd \end{cases}$$

TES^+ and TES^- can also be characterized by $\alpha = L + R$, and $\phi = \dfrac{R-L}{\alpha}$. Note that α represents the length of the circular interval. The sample path realizations generated by simulation using TES^+ and TES^- have shown discontinuity due to the crossing of the 0 point on the unit circular interval from both directions. For example, crossing clockwise will result in a jump from small values to large values. It was shown in [8] that a continuous sample path realization can be obtained by using a simple piece wise transformation T_ξ called stitching, where

$$T_\xi = \begin{cases} \frac{x}{\xi}, & x \in [0,\xi) \\ \frac{1-x}{1-\xi}, & x \in [\xi,1) \end{cases}$$

Autocorrelation of TES^+ and TES^-

The lag-1 auto-correlation for TES processes is derived in [2] and is given by

$$\rho_1^+(\alpha,\phi) = 1 - \frac{3+3\phi^2}{2}\alpha + \frac{1+3\phi^2}{2}\alpha^2 \tag{12}$$

$$\rho_1^-(\alpha,\phi) = -1 + \frac{3+3\phi^2}{2}\alpha - \frac{1+3\phi^2}{2}\alpha^2 \tag{13}$$

The value of α affects the magnitude of the correlation, while the value of ϕ affects the oscillating behavior of the auto-correlation. The larger the α, the smaller the magnitude. If $\phi = 0$, there will be no oscillation. For $\phi \uparrow 0$, the larger the ϕ, the faster the oscillation.

The recursive construction of the underlying TES processes is defined as follows

$$U_n^+ = \begin{cases} U_0, & n=0 \\ \langle U_{n-1}^+ + V_n \rangle, & n \rangle 0 \end{cases}$$

$$U_n^- = \begin{cases} U_n^+, & n \ even \\ 1-U_n^+, & n \ odd \end{cases}$$

Here, $\{V_n\}$ is a sequence of independent identically distributed random variables independent from U_0. The resulting sequences $\{U_n^+\}$ and $\{U_n^-\}$ are uniformly distributed in $[0,1)$ no matter what the density function of V_n, denoted as f_V. The choice of f_V will result in a different correlation structure of the resulting process.

The targeted sequences $\{X_n^+\}$ and $\{X_n^-\}$ are then obtained by using the inversions

$\{X_n^+\} = D(U_n^+)$ and $\{X_n^-\} = D(U_n^-)$

where $D = F^{-1}$ and F is marginal distribution of the desired sequence (the empirical data).

The fitting of the auto-correlation is done by a heuristic search for a pair ξ, f_V [1]. A combination of TES processes can also be used to better fit the auto-correlation. The empirical distribution is matched using the distribution inversion methods. The auto-correlation function of TES processes decays exponentially.

5. CONCLUSION

Traffic models are used in traffic engineering to predict network performance and to evaluate congestion control schemes. Traffic models vary in their ability to model various correlation structures and marginal distributions. Models that do not capture the statistical characteristics of the actual traffic result in poor network performance because they either over estimate, or under estimate the network performance. Traffic models must have a manageable number of parameters and the estimation of these parameters needs to be simple. Traffic models which are not analytically tractable can only be used to generate traffic traces. These traffic traces can be used in simulations.

REFERENCES

[1] FROST, V., MELAMED, B. Traffic modeling for telecommunications networks, *IEEE Communication magazine*, march 1994

[2] VOLNER, R. *Inteligence CATV – Traffic models, Design and Analysis*, International Conference on Computer, Communication and Control Technologies CCCT'03 and The 9th International Conference on Information Systems Analysis and Synthesis ISAS 03, Proceeding volume IV, July 2003, Orlando, Florida, USA, pp. 340 – 345, ISBN980-6560-05-1, CD - ISBN 980-6560-10-8

[3] VOLNER, R., BOREŠ, P., TICHÁ, D. *Long-range dependent traffic models CATV*, conference proceedings, The 8th Biennial Conference on Electronics and Microsystems Technology BEC 2002, Tallinn, Estonia, October 2002, pp. 347 – 350, ISBN 9985 – 59 – 292 – 1, poster

[4] VOLNER, R., BOREŠ, P., TICHÁ, D. *Regression models CATV*, conference proceedings, The 8th Biennial Conference on Electronics and Microsystems Technology BEC 2002, Tallinn, Estonia, October 2002, pp. 333 – 336, ISBN 9985 – 59 – 292 – 1

[5] VOLNER, R., BOREŠ, P., TICHÁ, D. *CATV and PC = Information Network*; proceedings of International Symposium on Signals, Circuits and Systems SCS '97, Iasi, Romania, October 1997, pp. 81 - 84

[6] VOLNER, R., BOREŠ, P. Aviation Data Networks, *Electronics and Electrical Engineering* N° 7 (63), Kaunas University of Technology, Academy of Sciences of Lithuana, Vilnius Gedimas Technical University, Riga Technical University, Tallinn Technical University, Kaunas 2005, pp. 22-26, ISSN 1392-1215

[7] VOLNER, R., SMRŽ, V. Security and Information Systems for Air Company, *IEEE Aerospace and Elektronic Systems magazine*, September 2007, Volume Twenty Two, Numer Nine, Part 1 of 2, pp. 23-27, ISSN 0885-8985, (in English), [50%],

[8] HAYES, M. *Statistical digital signal processing and modeling*, John Wiley, 1996

[9] Samorodnitsky, G., Taqqu, M. Stable non-Gaussian random processes, *Chapman and Hall*, 1994

[10] ADAS, A., MUKHERJEE, A. *On resource management and QoS guarantees for long range dependent traffic*, Proceeding IEEE Infocom 95, 1995

Reviewers: prof. Ing. Miloš Klíma, CSc.
Czech Technical University in Prague, Faculty of electrical engineering,
Department of Radioelectronics

prof. Ing. Zdeněk Žihla, Ph.D.
Business University of Prague, Department of Air Transport