MODELLING OF DYNAMIC BEHAVIOUR OF TRANSPORT MACHINES RANDOM EXCITED STRUCTURES

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Anotácia:Článok obsahuje stručnú charakteristiku stochastickej metódy, vhodnej k popisu dynamických systémov a modelovaniu ich správania sa v určených podmienkach. Metóda je založená na využití vektorových autoregresných modelov s kĺzavými priemermi tzv. ARMAV modelov. Obsahuje niektoré výsledky z experimentálneho overovania navrhnutého postupu, ktorý je možné využiť ako relatívne jednoduchý nástroj pre predikciu správania sa ľubovoľných stochastických systémov.

Kľúčové slová: mechanický dynamický systém, stochastické časové rady, autoregresný model

Summary:The paper is dealing with a newly developed method of identification of mechanical dynamic structures. This method is based on building statistically significant Vector Autoregresive Moving Average (ARMAV) models of random excited structures that define statistically significant modes of structure vibration. Once having ARMAV models of a structure it is very easy to simulate its behaviour by both stochastic and deterministic excitation in form of time series. The application is demonstrated on an example of Finite Element Model of simple structure using adaptive method of identification that can deal with non-stationary behaviour of structure also.

Key words: mechanical dynamic system, stochastic time series, autoregressive model.

1. INTRODUCTION

There are two commonly used methods for mechanical dynamic systems identification. The first of them, based on the drawing of structure, is the dynamic version of Finite Element Method. The second one is an experimental method based on determination of stochastic vibrations and mode shapes of structure using digital Fourier analyzer. The determination of modes and mode shapes of structures by means of FEM leads to the problem of eigenvalues (natural frequencies and damping rations). Usually same number of modes as the number of degrees of freedom is obtained. An effective method was developed [4], [7] to determine just such modes with statistical significant influence to the behaviour of structure.

By the experimental identification by means of digital Fourier analyzer there are two essential steps involved in the determination of the modal parameters and mode shapes. The starting point of the analysis technique relies on the estimation of spectra and subsequent determination of corresponding transfer functions.

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In the next step the measured transfer functions are used to extract the necessary modal information. A number of averaging, smoothing and estimation procedures are used in each step depending on the subjective judgment of analyst. Anticipating mostly the stochastic nature of mechanical dynamic system's excitation and response, time series methods and Data Dependent Systems (DDS) [3] approach seems to be very suitable and effective in the area of dynamic modelling too.

2. MATHEMATICAL FORMULATION AND ITS BACKGROUND

There are two theoretical areas concerning the above mentioned problems. The first is a classic approach to the vibrations of mechanical dynamic systems. It is well known, that in case of n-degree of freedom systems they are represented by system of ordinary differential equations of second order in matrix form as:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F(t)} \tag{1}$$

where **M**, **B** and **K** are (n x n) mass, damping and stiffness matrices, $\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$, $\mathbf{x}$ and **F(t)** are n-dimensional vectors of accelerations, velocities, displacements and force respectively. For the type of solution of this system are essential the eigenvalues of left-side of matrix equation, which are usually, obtained as complex numbers. The real parts of them have physical meaning as damping ratios and imaginary parts are natural frequencies.

The second theoretical area of theme is theory of stochastic processes. It was shown in [6], that any continuous system could be represented (in a scalar case) as differential equation in form:

$$\begin{aligned}\left(D^n + \alpha_{n-1}D^{n-1} + \alpha_{n-2}D^{n-2} + \ldots + \alpha_1 D^1 + \alpha_0\right).\,X(t) = \\ = \left(\beta_m D^m + \beta_{m-1}D^{m-1} + \ldots + \beta_1 D^1\right).\,Z(t)\end{aligned} \tag{2}$$

for which holds $E\left[Z(t)\right] = 0$ and $E\left[Z(t).Z(t-n)\right] = \delta(n)\,.\,\sigma_Z$ and where $\underline{n}$ and $\underline{m}$ indicate the order of the model, *D=d/dt* is the differential operator, *Z(t)* is white noise, *E* denotes the expectation operator, *δ(n)* is the Diracs delta function and α's and β's are model parameters. When such system is sampled at uniform interval Δt, differential equation (2) becomes a difference equation

$$X_t - a_1.X_{t-1} - a_2.X_{t-2} - \ldots - a_n.X_{t-n} = \varepsilon_t - b_1.\varepsilon_{t-1} - \ldots - b_{n-1}.\varepsilon_{t-n+1} \tag{3}$$

for which holds $E(\varepsilon_t) = 0$ and $E(\varepsilon_t.\varepsilon_{t-n}) = \delta_k.\sigma_e^2$, where X_t, X_{t-1}, X_{t-2}, … are values of process; *a*'s and *b*'s are parameters of the autoregressive model; ε_t, ε_{t-1}, ε_{t-2}, ..., ε_{t-n+1} are residuals; E denotes the expectation operator and δ_k is Kronecker delta function. Such a model is called **Autoregressive Moving Average of order (n,n-1) – ARMA (n,n-1).** It is obvious that parameters of continuous and discrete model of the same process must be functionally related.

The simplest way to express this relationship is by using roots μi and λi of characteristic equations of formula (2) and (3) respectively. The relationships then takes form as follows:

$$\lambda_i = e^{\mu_i \Delta t} \quad \text{or} \quad \mu_i = \frac{1}{\Delta t} \ln \lambda_i \tag{4}$$

In case of multivariate systems (involved mechanical dynamic systems, e.g. transport machines) the **Vector Autoregressive Moving Average Model - ARMAV** is obtained in form [2]:

$$\mathbf{A_0 . X_t - A_1 . X_{t-1} - \ldots - A_n . X_{t-n} = \varepsilon_t - D_1 . \varepsilon_{t-1} - \ldots - D_{n-1} . \varepsilon_{t-1}}$$

or in form (5)

$$\mathbf{(A_0 - A_1 . B^1 - A_2 . B^2 - \ldots - A_n . B^n) . X_t = (1 - D_1 . B^1 - D_2 . B^2 - \ldots - D_{n-1} . B^{n-1}) . \varepsilon_t}$$

which can fully express the relationships in structure during its vibrations and where $\mathbf{X_t}$ and $\boldsymbol{\varepsilon_t}$ are vectors of measurements and white noise series; $\mathbf{A_i}$ and $\mathbf{D_i}$ are matrices of system parameters; **B** is vector of back shift operators; $\boldsymbol{\sigma_{\varepsilon 2}}$ is matrix of dispersion and reciprocal correlation's and δ_k is Kronecker delta function.

3. THE GENERAL DESCRIPTION OF DYNAMIC SYSTEMS BEHAVIOUR

If one analyses a mechanical dynamic system with a numerical technique and its vibrations and exciting forces measure in uniform sampling intervals Δt, it is possible to develop discrete models to describe the relationship between output (vibration) and input (exciting forces) after Fig. 1.

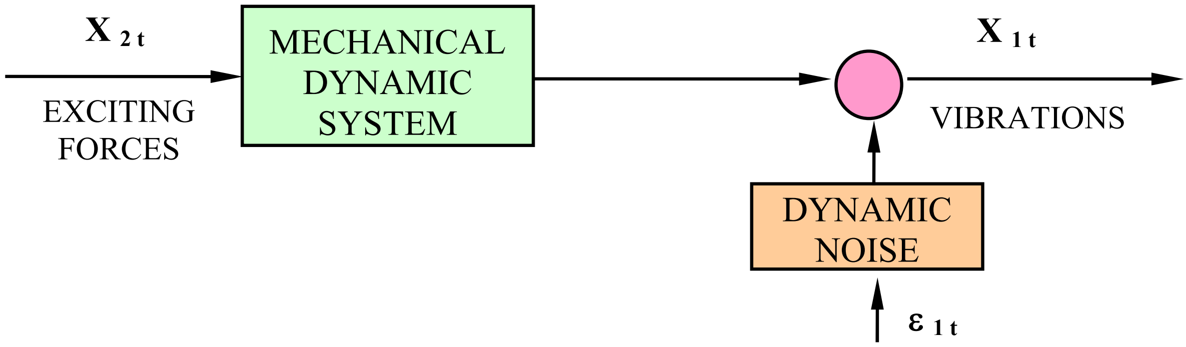

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Fig.1 - Block scheme of mechanical system dynamics

Dynamics of the mechanical system and dynamics of noise is represented by a discrete transfer function. Supposing non-existence of feed back between vibrations of structure can be expressed in its excitation (which hold for structures tests) one gets a resulting model of structure dynamics in form [3]

$$\left(1 - a_{111}.B - a_{112}.B^{2} - \ldots - a_{11n}.B^{n}\right).X_{1t} =$$
$$\left(a_{120} + a_{121}.B + \ldots + a_{12n}.B^{n}\right).X_{2t} + \tag{6}$$
$$+ \left(1 - b_{111}.B - b_{112}.B^{2} - \ldots - b_{11(n-1)}.B^{(n-1)}\right).\varepsilon_{1t}$$

where attached assumptions shown in formula (5).

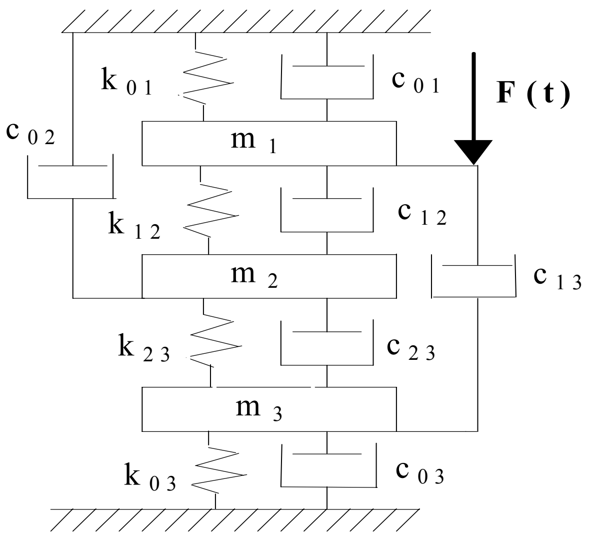
Procedure of statistically adequate models is getting concentrated in principle of output signals substituting (using non-linear least square method) with models of gradually increasing order until the decreased sum of squares becomes statistically non-significant on a chosen level of significance.

Physical meaning of such a procedure is in that we are trying to substitute the system with a model with the lowest number of statistically significant modes of vibrations. During this procedure, each increase of model order by two introduces (a further degree of freedom). If its contribution in not significant, the former model is taken as statistically adequate. In detail is involved strategy described in [4,5,6,7].

3.1. Modelling of Mechanical Dynamic System Behaviour

The theoretically correct approach mentioned in [2,7] and fully developed in [367] is to fit ARMA (2,1) model at first, to determine its parameters (by nonlinear least square method) and residual sum of squares. Then to increase the model orders by two and test the statistical significance of reduction of sum of squares by an F-criterion. Increasing of the model order by two continues until F-criterion shows insignificance of using higher order model.

As the examples of this procedure the results of identification of two simple model structures: three degrees of freedom - 3 DOF - system (Fig.2) and very simple beam structure (Fig.3) are shown in Tab.1 and Tab.2.



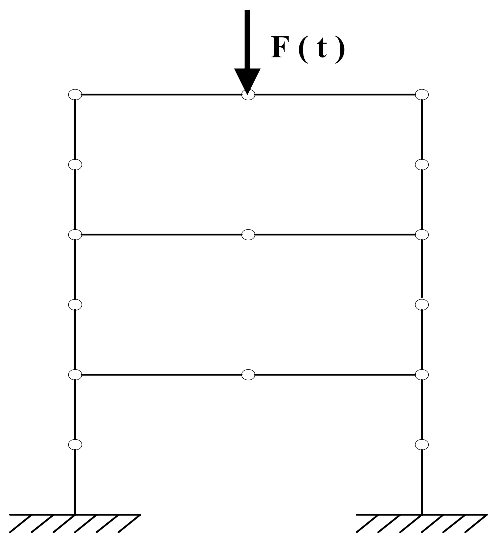


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Fig.2 - Three Degrees of Freedom System   Fig.3 - FEM Model (Beam Elements)

Parameters of identified ARMAV model

X	A_1	A_2	A_3	A_4	A_5	A_6
X_{1t}	1.5820	-2.8093	2.5325	-2.5329	1.2516	-0.7089
X_{2t}	1.3915	-2.4370	2.0704	-2.1175	-0.9362	-0.5819
X_{3t}	1.5859	-2.8081	2.4854	-2.4907	1.2183	-0.7271

Modal parameters of 3 DOF system identified

		1st Mode		2nd Mode		3rd Mode	
		n_1(Hz)	1	n_2(Hz)	2	n_3(Hz)	3
ARMAV	X_{1t}	1.2958	0.04701	1.9647	0.06908	2.7945	0.02754
	X_{2t}	1.2926	0.03468	2.1071	0.13760	2.8203	0.03408
	X_{3t}	1.2941	0.03069	1.9788	0.04885	2.8152	0.04163
Theoretical		1.2926	0.05289	2.0621	0.06664	2.8297	0.05945

Tab.1 - Summary of Identified Parameters and Modes 3 DOF System

	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode	6th Mode
FEM	8.406	25.873	42.439	103.935	126.626	126.626
ARMA	8.543	26.057	42.238	Non-sign.	Non-sign.	Non-sign.

Tab.2 - Natural Frequencies of Beam Structure

Above-mentioned procedure of identification using non-linear least square method and F-test of statistical adequacy is theoretically correct but very tedious and slow, therefore not suitable for on-line identification. For this purpose a new algorithm based on theory of adaptive and learning systems was developed and successfully tested in software ARMASA Package [1].

3.2. Simulation of Mechanical Dynamic System Behaviour

Once having the ARMA parameters of dynamic model determined, the procedure of dynamic performance of structure simulation is very simple. One has to start with generation of random residual εt, which are supposed to be normally distributed with zero mean and σε determined from residual sum of squares of the model.

The step-wise procedure looks like

1. *Generate* ε_1, *let* $X_1 = \varepsilon_1$,

2. *Generate* ε_2, *let* $X_2 = a_1.X_1 + \varepsilon_2 - b_1.\ \varepsilon_2$,

3. *Generate* ε_3, *let* $X_3 = a_1.X_2 + a_2.X_1 + \varepsilon_3 - b_1.\ \varepsilon_2 - b_2.\ \varepsilon_1$,

... .

(n+1) *Generate* ε_{n+1}, *let* $X_{n+1} = a_1.\ X_n + a_2.X_{n-1} + ... + a_n.\ X_1 + \varepsilon_{n+1} - b_1.\varepsilon_n - ... - b_{n-1}.\varepsilon_2$

(k>n+1) *Generate* ε_k, *let* $X_k = a_1.X_{k-1} + ... + a_n.X_{k-n} + \varepsilon k - b_1.\varepsilon_{k-1} - ... - b_{k-1}.\ \varepsilon_{k-n+1}$

The way of simulation of vector (multivariate) system is more complex but essentially the same. As an example, Fig.4 presents the results of stochastic vibrations simulation of 3 DOF system from Fig.2 by the use of parameters from Tab.1.

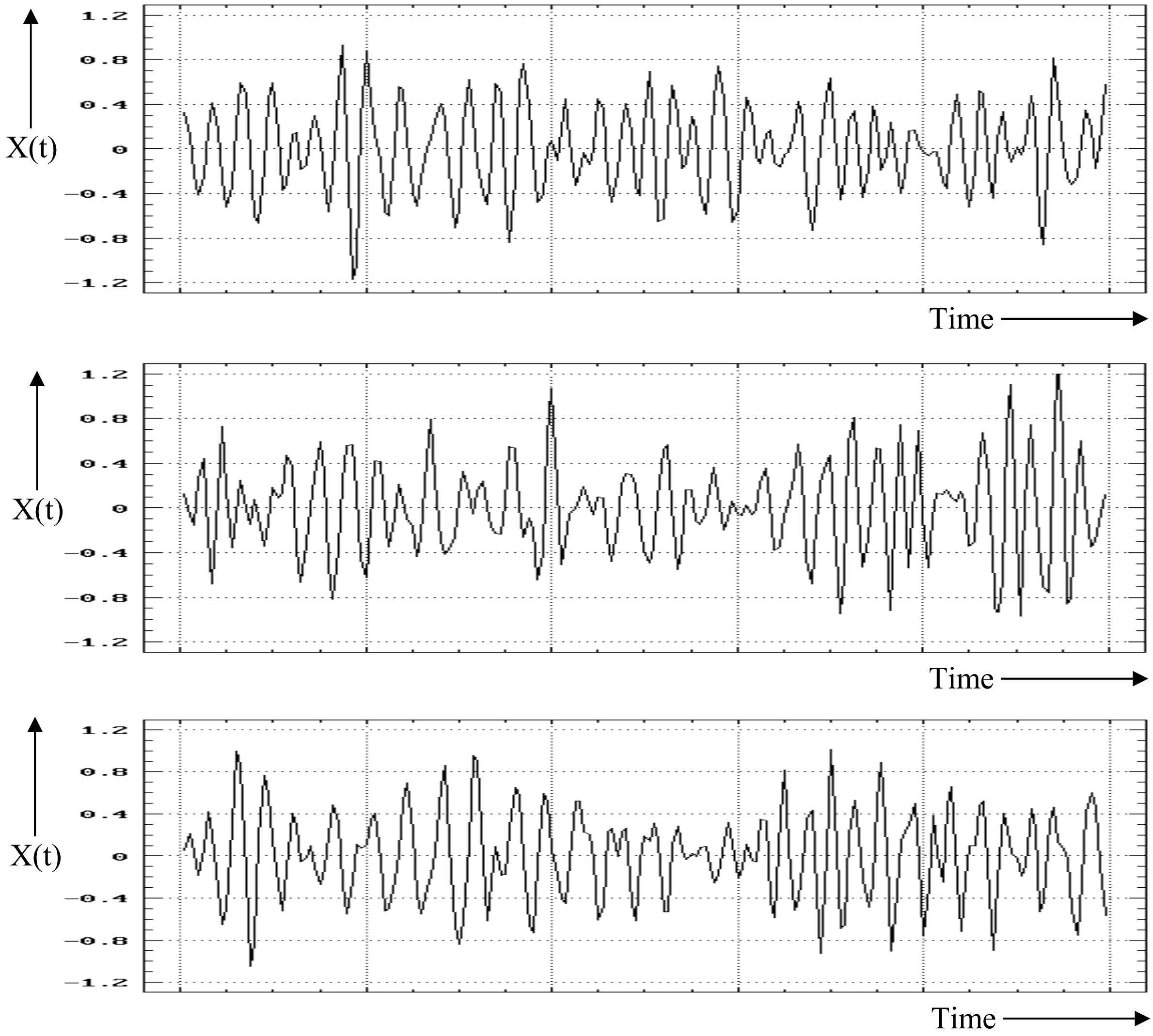

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Fig.4 - Results of dynamic behaviour modelling of 3 DOF system

4. CONCLUSIONS

It was namely a connection of proposed identification procedure with systems of complicated machine structures solution using Finite Elements Method. The advantage of using Autoregressive models consists of model parameters and modes that can be determined directly from these models not to be necessary to determine transfer functions. In addition, any subjective judgement is eliminated because statistic adequacy tests are exactly defined.

Results of further problems using proposed procedure by dynamic analysis and identification of modal characteristics of mechanical systems showed a relatively good agreement between theoretical and identified modes of vibrations, eigen-frequencies and relative damping. Advantage of ARMAV models utilisation is in fact that their parameters can one obtains directly from adequate models without necessity of transfer function determination. Further, any subjective judgement is eliminated because the tests of statistical adequacy are strictly defined.

From presented facts one can develop that above shown assumptions and theoretical starting points are correct and developed procedure can reduce number of calculation in an expressive way and improve efficiency of mechanical structures dynamic calculation. A relatively simple method for simulation of dynamic behaviour of statistically excited mechanical system was presented. The recently developed adaptive method for such system identification gives good assumptions for possible forecasting control of these systems.

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