KOMPENZACE PORUCH VZORKOVACÍ PERIODY V DISKRÉTNÍCH SYSTÉMECH ŘÍZENÍ LQG

HANDLING DISTURBANCES OF THE SCAN PERIOD IN DISCRETE LQG CONTROL SYSTEMS

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Anotace: Řídicí aplikace založené na využití počítačů, obzvláště pokud běží v prostředí běžných operačních systémů, často vykazují nekonstantnost vzorkovací periody zpracování vstupů a výstupů. Ačkoliv je tento jev obvykle zanedbáván, jsou-li využity diskrétní algoritmy řízení, v některých případech může způsobit výrazně horší výkonnost řídicí smyčky. V článku je navržen modifikovaný algoritmus řízení LQG, který bere v úvahu poruchy vzorkovací periody. Je rovněž ukázáno, že takovýto řídicí systém může být implementován i na poměrně jednoduchých hardwarových platformách.

Klíčová slova: LQG regulátor, stochastické řízení, hybridní systémy

Summary: Computer-based control applications, especially if they run under general-purpose operating systems, often exhibit variance of the scan period of processing inputs and outputs. Although this phenomenon is usually neglected when discrete control algorithms are used, in some cases it can cause significantly worse performance of the control loop. In the paper a modified discrete LQG control algorithm that takes disturbances of the scan period into account is proposed. It is also shown that such a controller can be implemented even on relatively simple hardware platforms..

Key words: LQG controller, stochastic control, hybrid systems

1. INTRODUCTION

In the control theory analysis and design of control algorithms in the continuous-time domain and in the discrete-time domain are treated separately. Continuous approach is natural for modelling and analysis of real objects and will be always used for designing controllers on the basis of analog components. Discrete approach seems to be advantageous for technical implementation of control algorithms on microprocessor-based platforms.

Discrete control algorithms rely upon constant period of processing inputs and outputs. However, constant scan period is usually not fully guaranteed in real situations. This phenomenon occurs usually due to handling asynchronous hardware events in computer systems. Although this problem is typical for general-purpose multitasking operating systems,

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even the most robust hardware platforms such as PLCs exhibit scan variance, especially if these systems communicate over a network.

Irregularities of the scan period cause worse performance of the control loop in comparison to theoretical case. This influence is can be neglected if they occur rarely and the system time constants are large in comparison to the scan period. In other cases the influence on the closed loop dynamics can be significant.

In this paper we describe a modification of the stochastic linear-quadratic-Gaussian (LQG) discrete control algorithm taking into account irregularities of the scan period. We show that the effect of the scan variance can be partially compensated by mathematical means if a hybrid control law is used, working at discrete steps but using a continuous-time model for the state estimation and for determination of the control output.

2. PROBLEM FORMULATION

Consider a continuous time-invariant linear system

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{F}\mathbf{x}(t) + \mathbf{G}\mathbf{u}(t) + \mathbf{L}\mathbf{w}(t) \\ \mathbf{y} &= \mathbf{H}\mathbf{x}(t) + \mathbf{n}(t) \end{aligned} \tag{1}$$

where dimensions of **x** and **u** are *n* and *m*, respectively, $n \geq m$. **F**, **G**, **H**, and **L** are known matrices of corresponding dimensions. Disturbance input $\mathbf{w}(t)$ and the measurement noise $\mathbf{n}(t)$ are uncorrelated white Gaussian random processes with zero mean value and autocorrelations

$$\begin{aligned} E\left\{\mathbf{w}(t)\mathbf{w}^T(\tau)\right\} &= \mathbf{W}\delta(t-\tau) \\ E\left\{\mathbf{n}(t)\mathbf{n}^T(\tau)\right\} &= \mathbf{N}\delta(t-\tau) \end{aligned} \tag{2}$$

where $\delta(t)$ is Dirac impulse function. We are looking for a control history such that

$$J = \frac{1}{2} E\left\{ \int_0^{t_f} \mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)\, dt \right\} \to \min \tag{3}$$

where $E\{.\}$ is the mean value operator, **Q**, **R** are given symmetric positive definite matrices and t_f is a large enough constant.

We assume that the current state $\mathbf{x}(t)$ is not known, but has to be estimated from the measurements $\mathbf{y}(t)$. It is known that linear-quadratic control problems with Gaussian disturbances obey a *separation property*, which enables to design the control and estimation logic separately and moreover they possess the *certainty-equivalence property*, which means that the optimal controller is the same as in deterministic case without disturbances [2] – [4].

Although the system nature is continuous, we consider that the measurements are taken at discrete time steps $t_0 < t_1 < \ldots$. We assume that the scan instants are not equidistant, but the instants of past measurements are known. Thus each measurement $\mathbf{y}(t_k)$ is provided with its time mark. This requirement usually can be easily technically realized. The same instants are used for generating control output. We assume that the control output generated at time t_i is constant until the next scan at t_{i+1}. If the current scan time t_k is known, the next scan instants $t_{k+1}, t_{k+2}, \ldots$ are considered as random quantities with known expected values $\overline{t}_{k+1}, \overline{t}_{k+2}, \ldots$. Note that t_{k+1} usually does not depend on the current value of t_k, since it is derived from global system clock.

Denote $\{\hat{t}_i\}$ known instants of the hardware clock interrupts. It holds

$$\hat{t}_{i+1} - \hat{t}_i = T = const. \tag{4}$$

We adopt the following simple model for the sequence $\overline{t}_{k+1}, \overline{t}_{k+2}, \ldots$, which corresponds to many real platforms:

$$\begin{aligned} \overline{t}_{i+1} &= \overline{t}_i + T, \quad i \geq k+1 \\ \overline{t}_{k+1} &= \hat{t}_n \end{aligned} \tag{5}$$

where $\hat{t}_n$ is the least interrupt instant after t_k such that

$$\hat{t}_n > t_k + d \tag{6}$$

and $d \in (0, T)$ is a known parameter. In this model a significant scan delay can cause loosing one scan instant and the values $\overline{t}_{k+1}, \overline{t}_{k+2}, \ldots$ (expected future scan instants) are equidistant.

3. OPTIMAL STATE ESTIMATION

Consider that the optimal state estimate $\hat{\mathbf{x}}_{k-1} = \hat{\mathbf{x}}(t_{k-1})$ and the state covariance estimate

$$\hat{\mathbf{P}}_{k-1} = \hat{\mathbf{P}}(t_{k-1}) = E\left\{\left(\mathbf{x}_{k-1} - \hat{\mathbf{x}}_{k-1}\right)\left(\mathbf{x}_{k-1} - \hat{\mathbf{x}}_{k-1}\right)^T\right\} \tag{7}$$

are known at time t_{k-1}. The state estimate can be extrapolated from t_{k-1} to t_k. By integrating equations (1) and using $\hat{\mathbf{x}}_{k-1}$ as initial condition we have

$$\begin{aligned} \overline{\mathbf{x}}_k &= \mathbf{\Phi}(t_k, t_{k-1})\hat{\mathbf{x}}_{k-1}(t) + E\left\{\int_{t_{k-1}}^{t_k} \mathbf{\Phi}(t_k, \tau)\left[\mathbf{G}\mathbf{u}(\tau) + \mathbf{L}\mathbf{w}(\tau)\right] d\tau\right\} = \\ &= \mathbf{\Phi}(t_k, t_{k-1})\hat{\mathbf{x}}_{k-1}(t) + \int_{t_{k-1}}^{t_k} \mathbf{\Phi}(t_k, \tau)\, d\tau . \mathbf{G}\mathbf{u}_{k-1} = \mathbf{\Phi}(t_k, t_{k-1})\hat{\mathbf{x}}_{k-1}(t) + \mathbf{\Psi}(t_k, t_{k-1})\,\mathbf{u}_{k-1} \end{aligned} \tag{8}$$

where $\mathbf{\Phi}(t_k, t_{k-1})$ is the transition matrix from t_{k-1} to t_k , $\mathbf{u}_{k-1}$ is the control value set at t_{k-1}, which is constant in $\left[t_{k-1}, t_k\right)$. Since the system is time-invariant, the matrices $\mathbf{\Phi}(t, t_k)$ and $\mathbf{\Psi}(t, t_k)$ depend only on difference $t - t_k$, i.e. $\mathbf{\Phi}(t, t_k) \equiv \mathbf{\Phi}(t - t_k)$ and $\mathbf{\Psi}(t, t_k) \equiv \mathbf{\Psi}(t - t_k)$. Since holds

$$\mathbf{x}_k - \overline{\mathbf{x}}_k = \mathbf{\Phi}(t_k, t_{k-1})(\mathbf{x}_{k-1} - \hat{\mathbf{x}}_{k-1}) + \int_{t_{k-1}}^{t_k} \mathbf{\Phi}(t_k, \tau)\mathbf{L}\mathbf{w}(\tau)\, d\tau \tag{9}$$

for the covariance extrapolation $\overline{\mathbf{P}}_k$ we easily obtain

$$\overline{\mathbf{P}}_k = \mathbf{\Phi}(t_k, t_{k-1})\hat{\mathbf{P}}_{k-1}\mathbf{\Phi}^T(t_k, t_{k-1}) + \mathbf{\Gamma}(t_k, t_{k-1}) \tag{10}$$

where

$$\mathbf{\Gamma}(t_k, t_{k-1}) = \int_{t_{k-1}}^{t_k}\int_{t_{k-1}}^{t_k} \mathbf{\Phi}(t_k, \tau)\mathbf{L}E\left\{\mathbf{w}(\tau)\mathbf{w}^T(\alpha)\right\}\mathbf{L}^T\mathbf{\Phi}^T(t_k, \alpha)\, d\alpha\, d\tau$$

$$= \int_{t_{k-1}}^{t_k} \mathbf{\Phi}(t_k, \tau)\mathbf{L}\mathbf{W}\mathbf{L}^T\mathbf{\Phi}^T(t_k, \tau)\, d\tau\ . \tag{11}$$

Optimal-variance estimate $\hat{\mathbf{x}}_k$ of $\mathbf{x}_k$ is provided by the Kalman filter formula [1], [3]:

$$\begin{aligned}\hat{\mathbf{x}}_k &= \mathbf{\Phi}(t_k - t_{k-1})\hat{\mathbf{x}}_{k-1} + \mathbf{\Psi}(t_k - t_{k-1})\,\mathbf{u}_{k-1} + \\ &\quad + \mathbf{K}_k\left[\mathbf{y}(t_k) - \mathbf{H}\left(\mathbf{\Phi}(t_k - t_{k-1})\hat{\mathbf{x}}_{k-1} + \mathbf{\Psi}(t_k - t_{k-1})\,\mathbf{u}_{k-1}\right)\right]\end{aligned} \tag{12}$$

where the gain matrix $\mathbf{K}_k$ is given by

$$\mathbf{K}_k = \overline{\mathbf{P}}_k\mathbf{H}^T\left[\mathbf{H}\overline{\mathbf{P}}_k\mathbf{H}^T + \mathbf{N}\right]^{-1} \tag{13}$$

and the covariance estimate update for the next step is

$$\hat{\mathbf{P}}_k = \left[\overline{\mathbf{P}}_k^{-1} + \mathbf{H}^T\mathbf{N}^{-1}\mathbf{H}\right]^{-1} = (\mathbf{I}_n - \mathbf{K}_k\mathbf{H})\overline{\mathbf{P}}_k\ . \tag{14}$$

4. OPTIMAL CONTROL ALGORITHM

We adopt the model (5), (6) for prediction of the scan instants t_i, $i > k$. The current time t_k and the current state estimate $\hat{\mathbf{x}}_k$ are known. For simplicity we denote t_i the estimates of t_i for $i \geq k+1$, instead of $\overline{t_i}$. The criterion value from t_k to t_f can be expressed as

$$J(t_k) = \frac{1}{2}E\left\{\sum_{i=k}^{N-1}\int_{t_i}^{t_{i+1}} \mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)\, dt\right\} \tag{15}$$

where N is a sufficiently large integer. Note that unlike common practice the criterion includes information about complete state history in $\left[0, t_f\right]$, and not only about values at discrete points $\{t_i\}$.

Denote for simplicity $\overline{\mathbf{x}}_i = E\{\mathbf{x}(t_i)\}$, $\mathbf{u}_i = \mathbf{u}(t_i)$. For $t \in [t_i, t_{i+1})$ holds

$$\mathbf{x}(t) = \mathbf{\Phi}(t, t_i)\mathbf{x}(t_i) + \mathbf{\Psi}(t, t_i)\mathbf{u}_i + \int_{t_i}^{t} \mathbf{\Phi}(t, \tau)\mathbf{L}\mathbf{w}(\tau)\, d\tau . \tag{16}$$

By substituting into (15) and using the fact that $\overline{\mathbf{x}}_k = \hat{\mathbf{x}}_k$, we obtained

$$J(t_k) = \frac{1}{2}\left[\hat{\mathbf{x}}_k^T \ \mathbf{u}_k^T\right]\mathbf{U}(t_{k+1} - t_k)\begin{bmatrix}\hat{\mathbf{x}}_k \\ \mathbf{u}_k\end{bmatrix} + \frac{1}{2}\Xi(t_{k+1} - t_k) + J(t_{k+1}) \tag{17}$$

where

$$J(t_{k+1}) = \frac{1}{2}\sum_{i=k+1}^{N-1}\left[\overline{\mathbf{x}}_i^T \ \mathbf{u}_i^T\right]\mathbf{U}(T)\begin{bmatrix}\overline{\mathbf{x}}_i \\ \mathbf{u}_i\end{bmatrix} + \frac{N-k-1}{2}\,\Xi(T) \tag{18}$$

and

$$\Xi(h) = \int_0^h \int_0^h \mathrm{Tr}\left(\mathbf{Q}\mathbf{\Phi}(t, \tau)\mathbf{L}\mathbf{W}\mathbf{L}^T\mathbf{\Phi}^T(t, \tau)\right) d\tau\, dt . \tag{19}$$

The matrix function $\mathbf{U}(h)$ has the form

$$\mathbf{U}(h) = \begin{bmatrix}\tilde{\mathbf{Q}}(h) & \tilde{\mathbf{M}}(h) \\ \tilde{\mathbf{M}}^T(h) & \tilde{\mathbf{R}}(h)\end{bmatrix} \tag{20}$$

where

$$\tilde{\mathbf{Q}}(h) = \int_0^h \mathbf{\Phi}^T(\tau)\mathbf{Q}\,\mathbf{\Phi}(\tau)\,d\tau , \ \tilde{\mathbf{M}}(h) = \int_0^h \mathbf{\Phi}^T(\tau)\mathbf{Q}\,\mathbf{\Psi}(\tau)\,d\tau \ ,$$

$$\tilde{\mathbf{R}}(h) = \int_0^h \left(\mathbf{\Psi}^T(\tau)\mathbf{Q}\mathbf{\Psi}(\tau) + \mathbf{R}\right)d\tau . \tag{21}$$

Denote $J^*(t_k)$ the minimal value of $J(t_k)$. Since the stochastic terms $\Xi(t_{k+1} - t_k)$, $\Xi(T)$ do not contain control and state, they do not take part in the minimization of $J(t_k)$. By application of the Bellman optimality principle [1] we obtain

$$\begin{aligned} J^*(t_k) = \min_{u_k}&\left\{\frac{1}{2}\left[\hat{\mathbf{x}}_k^T \ \mathbf{u}_k^T\right]\mathbf{U}(t_{k+1} - t_k)\begin{bmatrix}\hat{\mathbf{x}}_k \\ \mathbf{u}_k\end{bmatrix} + V_{k+1}^*\right\} \\ &+ \frac{1}{2}\left[\Xi(t_{k+1} - t_k) + (N-k-1)\Xi(T)\right] \end{aligned} \tag{22}$$

where

$$V_{k+1} = \frac{1}{2}\sum_{i=k+1}^{N-1}\begin{bmatrix}\bar{\mathbf{x}}_i^T & \mathbf{u}_i^T\end{bmatrix}\mathbf{U}(T)\begin{bmatrix}\bar{\mathbf{x}}_i \\ \mathbf{u}_i\end{bmatrix} \tag{23}$$

and

$$V_{k+1}^* = \min_{\{u_{k+1},\dots,u_{N-1}\}}\left\{V_{k+1}\right\}. \tag{24}$$

Minimization of V_{k+1} with dynamic constraints

$$\bar{\mathbf{x}}_{i+1} = \begin{bmatrix}\mathbf{\Phi}(T), & \mathbf{\Psi}(T)\end{bmatrix}\begin{bmatrix}\bar{\mathbf{x}}_i \\ \mathbf{u}_i\end{bmatrix} \tag{25}$$

is a discrete deterministic linear-quadratic optimal control problem (the instants $t_{k+1}, t_{k+2}, \ldots$ are equidistant by assumption). The optimal cost-function value for $N \to \infty$ is in the form

$$V_{k+1}^* = \frac{1}{2}\bar{\mathbf{x}}_{k+1}^T\mathbf{S}\,\bar{\mathbf{x}}_{k+1} = \frac{1}{2}\begin{bmatrix}\hat{\mathbf{x}}_k^T, & \mathbf{u}_k^T\end{bmatrix}\begin{bmatrix}\mathbf{\Phi}^T(h) \\ \mathbf{\Psi}^T(h)\end{bmatrix}\mathbf{S}\begin{bmatrix}\mathbf{\Phi}(h), \mathbf{\Psi}(h)\end{bmatrix}\begin{bmatrix}\hat{\mathbf{x}}_k \\ \mathbf{u}_k\end{bmatrix} \tag{26}$$

where $h = t_{k+1} - t_k$ is expected distance of the next scan and $\mathbf{S}$ is determined as a steady-state solution of the difference Riccati equation with terminal condition $\mathbf{S}_N = 0$ [3]:

$$\begin{aligned}\mathbf{S}_i &= \tilde{\mathbf{Q}}(T) + \mathbf{\Phi}^T(T)\mathbf{S}_{i+1}\mathbf{\Phi}(T) - \\ &-\left(\tilde{\mathbf{M}}(T) + \mathbf{\Phi}^T(T)\mathbf{S}_{i+1}\mathbf{\Psi}(T)\right)\left(\tilde{\mathbf{R}}(T) + \mathbf{\Psi}^T(T)\mathbf{S}_{i+1}\mathbf{\Psi}(T)\ \right)^{-1}\left(\tilde{\mathbf{M}}^T(T) + \mathbf{\Psi}^T(T)\mathbf{S}_{i+1}\mathbf{\Phi}(T)\right).\end{aligned} \tag{27}$$

The minimizer of $J(t_k)$ is

$$\mathbf{u}_k^* = \arg\min_{\mathbf{u}_k}\left\{\frac{1}{2}\begin{bmatrix}\hat{\mathbf{x}}_k^T & \mathbf{u}_k^T\end{bmatrix}\mathbf{Z}(h)\begin{bmatrix}\hat{\mathbf{x}}_k \\ \mathbf{u}_k\end{bmatrix}\right\} \tag{28}$$

where

$$\mathbf{Z}(h) = \mathbf{U}(h) + \begin{bmatrix}\mathbf{\Phi}^T(h)\mathbf{S}\mathbf{\Phi}(h) & \mathbf{\Phi}^T(h)\mathbf{S}\mathbf{\Psi}(h) \\ \mathbf{\Psi}^T(h)\mathbf{S}\mathbf{\Phi}(h) & \mathbf{\Psi}^T(h)\mathbf{S}\mathbf{\Psi}_k(h)\end{bmatrix}. \tag{29}$$

The solution easily found by differentiation is in the form

$$\mathbf{u}_k^* = -\mathbf{E}(h)^{-1}\mathbf{D}(h)\hat{\mathbf{x}}_k \tag{30}$$

where

$$\mathbf{D}(h) = \tilde{\mathbf{M}}^T(h) + \mathbf{\Psi}^T(h)\mathbf{S}\mathbf{\Phi}(h), \quad \mathbf{E}(h) = \tilde{\mathbf{R}}(h) + \mathbf{\Psi}^T(h)\mathbf{S}\mathbf{\Psi}(h). \tag{31}$$

Note that the matrix inversion in (30) exists, since $\tilde{\mathbf{R}}$ and $\mathbf{S}$ are positive definite. The controller matrix $\mathbf{C}(h)$ is dependent on expected distance of the next scan *h*. Since the stochastic terms do not take part in minimization, the certainty equivalence property holds in this case as well.

5. IMPLEMENTATION OF THE CONTROLLER

Obtained expressions are rather complicated to be computed at each control step. The steady-state solution of the Riccati equation (27) is constant and can be obtained off-line as a part of the controller design. Basic method of obtaining **S** consists in solving (27) iteratively until $\|\mathbf{S}_{i+1}-\mathbf{S}_i\|<\varepsilon$. More efficient methods are proposed in [5], [6].

If we assume that it is guaranteed for some $h_{\max}>0$

$$t_{i+1}-t_i<h_{\max} \tag{32}$$

the values of needed matrix functions $\mathbf{\Phi}(h),\mathbf{\Psi}(h),\mathbf{\Gamma}(h)\,,\tilde{\mathbf{Q}}(h),\tilde{\mathbf{M}}(h),\tilde{\mathbf{R}}(h)$ can be computed for h from 0 to $h_{\max}$ with a given step Δh and stored in computer memory. In on-line mode it is possible only to select values corresponding to a nearest value of h_i .

The transition matrix can be computed directly by integrating the definition equations

$$\frac{d}{dh}\mathbf{\Phi}(h)=\mathbf{F}\mathbf{\Phi}(h),\quad \mathbf{\Phi}(0)=\mathbf{I}_n\,. \tag{33}$$

The expressions for the other matrices dependent on h can be rewritten into the form of differential equations as well. Formally written, it is needed to integrate the following set of equations:

$$\frac{d}{dh}\begin{bmatrix}\mathbf{\Phi}(h)\\ \mathbf{\Psi}(h)\\ \mathbf{\Gamma}(h)\\ \tilde{\mathbf{Q}}(h)\\ \tilde{\mathbf{M}}(h)\\ \tilde{\mathbf{R}}(h)\end{bmatrix}=\begin{bmatrix}\mathbf{F}\mathbf{\Phi}(h)\\ \mathbf{\Phi}(h)\mathbf{G}\\ \mathbf{\Phi}(h)\mathbf{L}\mathbf{W}\mathbf{L}^T\mathbf{\Phi}^T(h)\\ \mathbf{\Phi}^T(h)\mathbf{Q}\mathbf{\Phi}(h)\\ \mathbf{\Phi}(h)\mathbf{Q}\mathbf{\Psi}(h)\\ \mathbf{\Psi}^T(h)\mathbf{Q}\mathbf{\Psi}(h)+\mathbf{R}\end{bmatrix},\quad \begin{bmatrix}\mathbf{\Phi}(0)\\ \mathbf{\Psi}(0)\\ \mathbf{\Gamma}(0)\\ \tilde{\mathbf{Q}}(0)\\ \tilde{\mathbf{M}}(0)\\ \tilde{\mathbf{R}}(0)\end{bmatrix}=\begin{bmatrix}\mathbf{I}_n\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}\,. \tag{34}$$

Computation of matrix inversions in (13) and (30) in real time may be disadvantageous. However, the controller matrix $\mathbf{C}(h)$ can be pre-computed on the basis of the solution of (34). To avoid matrix inversion in the Kalman filter formula (13) it is possible to process individual components of the measurement vector **y** separately [3].

The results were tested for the plant

$$\mathbf{F} = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ \ \mathbf{H} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ \ \mathbf{L} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \mathbf{I}_2 \tag{35}$$

with initial conditions

$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ \hat{\mathbf{x}}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \ \mathbf{P}_0 = \mathbf{I}_2 . \tag{36}$$

The system is unstable and oscillating. Assumed noise covariances are $\mathbf{W} = \mathbf{I}_2$, $\mathbf{N} = [1]$, but noise inputs are kept zero in simulation. The criterion parameters were chosen as

$$\mathbf{Q} = \mathbf{I}_2 , \ \ \mathbf{R} = \mathbf{I}_2 . \tag{37}$$

The scan period is $T = 0.5\,s$, but each third scan was delayed of 50%. Figures 1-2 show response of the system and of the control variable for classical discrete LQG controller tuned for constant $T = 0.5\,s$. Figures 3-4 show corresponding histories for the hybrid controller given by equations (12) and (30). For $T = 1\,s$ we even obtained non-stable behaviour of classical controller, but hybrid controller was still able to stabilize the system rather efficiently. It is however true, that under normal conditions the scan period is usually more regular and shorter in comparison to the system time constants. The differences then may be unimportant.

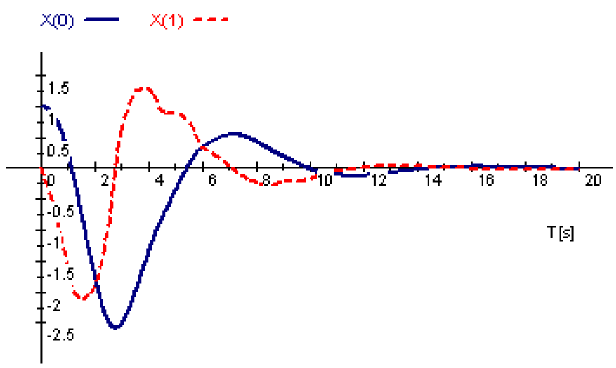

Fig. 1: History of the state – discrete controller

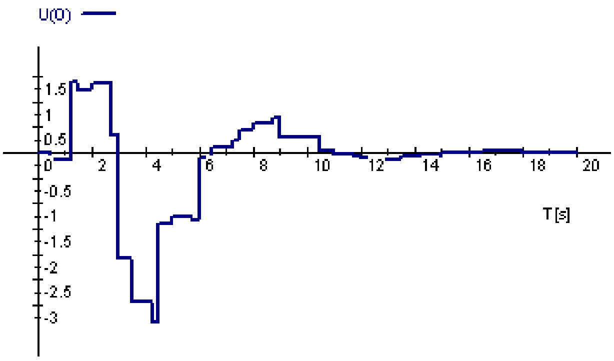

Fig. 2: History of the control variable – discrete controller

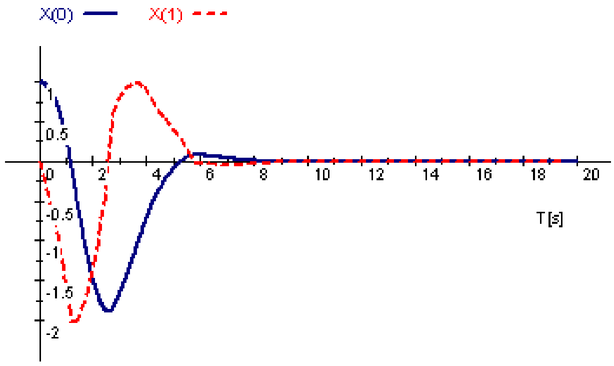

Fig. 3: History of the state - hybrid controller

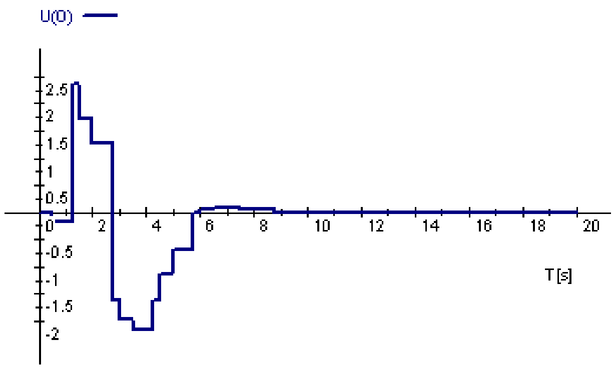

Fig. 4: History of the control variable - hybrid controller

6. CONCLUSION

The modification of the LQG control algorithm described in this paper reduces influence of the scan period variance on the closed-loop performance. This modification regards both the state estimation and the control law. Although obtained expressions are rather complicated to be computed in real time, if a sufficient memory in the control system is available, it is possible to carry out most of the computations in forward and the control action itself is not a complicated or a time-consuming operation. Consequently, such a control law can be implemented even on relatively simple hardware platforms.

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