RIEŠENIE ÚLOHY VRCHOLOVÉHO POKRYTIA S RÔZNYMI CENAMI VRCHOLOV

ON VERTEX COVER PROBLEM WITH THE UNIQUE PRICE OF VERTICES

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Anotácia: Tento článok predkladá algoritmus heuristického riešenia úlohy vrcholového pokrytia s rôznymi cenami vrcholov. Tento algoritmus pozostáva z dvoch algoritmov.

Prvý algoritmus prehľadáva vrcholy od maximálneho stupňa zostupne a cenu vybraného vrcholu porovnáva so súčtom cien susedných vrcholov. Ak je cena vybraného vrcholu menšia nanajvýš rovná súčtu cien susedných vrcholov, potom tento vrchol vložíme do hľadanej minimálnej množiny vrcholov. V opačnom prípade vkladáme do minimálnej množiny susedné vrcholy.

Druhý algoritmus je založený na vylučovaní zbytočných vrcholov z minimálnej množiny vrcholov, ktorú sme získali po prvom algoritme.

Kľúčové slova: maximálna cena, minimálna cena, najvyšší stupeň, najnižší stupeň, vrcholové pokrytie, susedný vrchol, špecifická cena, zbytočný vrchol.

Summary: This paper presents algorithm of the heuristic solution of vertex cover with the unique price of vertices. This algorithm consists of two algorithms.

The first algorithm scans vertices from a maximal degree decreasingly and compares price of chosen vertex with the sum of prices of neighboring vertices.

If the price of chosen vertex is less than or equal to the sum of prices of neighboring vertices than we place this vertex into a optimal solution. In opposite case we place neighboring vertices into the optimal solution. We repeat this process until it is possible with respekt to conditions.

The second algorithm segregates superfluous vertices from the optimal set after the first algorithm.

Key words: biggest degrees, lowest degrees, maximal price, minimal price, neighboring vertex, superfluous vertex, vertex cover, unique price

1 INTRODUCTION

In practice there are many problems that can be described following way. Let G be a graph that consist of set of vertices V and set of edges E. Every edge is represented by a pair of vertices from V. Every vertex has its unique value that is that is the price of the vertex.

The problem is as follows:

At first we will find a subset V‘ of the set of vertices V, such that every edge of E will coincide with at lest one vertex of V‘.

The second condition is that the sum of vertices in V‘ is minimal.

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This paper deals with one problem from [1] which is stated in the following form.

2 VERTEX COVER PROBLEM WITH THE UNIQUE PRICE OF VERTICES VCPUPV

Instance. Graph $G=(V,E)$, positive integers $K \leq |V|$ and P

Question. Is there a vertex cover of size k or less for G, i.e. a subset $V' \subseteq V$, with $|V'| \leq K$ such that for each edge $\{u,v\} \in E$ at least one of u and v belongs to V' and *sum* $c_{i1}+c_{i2}+\ldots+c_{ik} < P$?

Let a_{ji} are coefficients of incidention of elements q_j of the set M with the sets M_i, $j = 1,2,\ldots,m$, $i = 1,2,\ldots,n$, i.e. the row $a_{j1}, a_{j2}, \ldots, a_{jn}$ describes the occurrence of the point q_j in the sets $M_1, M_2, \ldots, M_n$, i.e.

$$a_{ji} = 1 \quad \text{if} \quad q_i \in M_j$$

$$a_{ji} = 0 \quad \text{if} \quad q_i \notin M_j$$

VCPUPV is the problem of the covering of the m rows of the 0–1 matrix $\lfloor a_{ji} \rfloor$ by the subset of the n columns with respect to the minimal price.

We define $y_i = 1$ if the column i $(c_i \rangle 0)$ belongs to the solution

$y_i = 0$ otherwise

$$\min \quad c_1y_1 + c_2y_2 + \ldots + c_ny_n$$

subject to:

$$a_{11}y_1 + a_{12}y_2 + \ldots + a_{1n}y_n \geq 1$$

$$a_{21}y_1 + a_{22}y_2 + \ldots + a_{2n}y_n \geq 1$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_{m1}y_1 + a_{m2}y_2 + \ldots + a_{mn}y_n \geq 1,$$

$$y_i \in \{0,1\}$$

2.1 The algorithm strategy

Similarly as in the algorithm for SCP [4] we will choose the vertices decreasingly according to the degree of the vertex v into the optimal set V' from the set V. The difference between VCP and VCPUPV is the vertices level. Vertices in the VCP are chosen arbitrary of the level r. In the VCPUPV we will recognize the vertices from the minimal price to the maximal.

We will create a set V2 of vertices of a degree r. In this set we will choose vertices from the minimal price to the maximal price of the vertices.

If a vertex v of degree r has a property that the price of vertex v is less than or equal to the sum of the prices its neighbouring vertices then we put the vertex v to the optimal set V'.

If the vertex price is greater than the sum of the prices of the neighbouring vertices, then we do not choose the vertex into the set V'.

This process is illustrated in the description of algorithm and it is described in the algorithm.

Comments. V, V', V'' are sets of vertices of a graph G and E is a set of edges of this graph G. Now we give one algorithm of solution of Vertex cover problem with the unique price of vertices.

2.2 Description of the algorithm

We describe the algorithm by the two ways, i.e. two algorithms.

In the 1-st algorithm we will construct the set V' by the following way. It consists of the vertices of the biggest degree and the minimal price consequently.

Step 0: We take $r = r_{max}$, $c_{min} \leq c \leq c_{max}$, where $v_{rmax,\ cmin}$ are vertices with the maximal degree and the minimal price.
Step 1: We construct the set V_2 which contains all vertices of degree r and of price c.
Step 2: We choose arbitrary vertex $v \in V_2$, and then we place into V'.

By this way we covered all edges e_i coincidating with the vertex v. For any vertex v_i, $v_i \neq v$ coinciding with edge e_i we decrease the degree of this vertex of one degree. If some vertices v_i in V_2 then we assume (according to degraded of the degree) to be excluded. The step 2 will repeat with all vertices v until V_2 is empty.

If V_2 is empty, then we increase the price c, i.e. we take $c = c + 1$ and then we go to the step 1.
If $c > c\ max$, then we take $r = r - 1$ and we go to the step 1.
Step 3: This algorithm we repeat until every edge is covered.

In the 2-nd algorithm we will construct the set V'' by the following way.

We will exclude superfluous vertices after the 1-st algorithm from the optimal set V' similarly as the 1-st algorithm but we will start from below (r = rmin, c = cmin) above (r = rmax, c = cmax) consequently. In this we will construct the set V'' as the optimal solution to the problem.

Next we will present algorithm 1 an algorithm 2 consequently.

3 ALGORITHMS

3.1 Algorithm for vertex cover with the price of vertices:(insert the components - 1-st algorithm)

read V, V is the set of vertices of the graph.
read E, E is the set of edges of the graph.
$V_1 = V, E_1 = E$ (we create the copies of these sets).
find $r\max, v_{r\max}$ are the vertices of the maximal degree.
find $c_{\min}, c_{\max}$ are the maximal and minimal price of the vertices.
put $r = r\max, c = c_{\min}, V' = \emptyset$ (V' is a created locked optimal set of vertices), $V_3 = \emptyset$

1. put $V_2, V_2 = \{v, v \in V_1, d(v) = r, p(v) = c\}$, i.e. V_2 is a set of vertices of degree r

if $V_2 = \emptyset$, $c < c_{\max}$, go to 3, else go to 4.

2. if $|V_2 \cap V_3| > 0$, choose arbitrary $v \in V_2 \cap V_3$, else
   choose arbitrary $v \in V_2$

   $E_2 = \{e_i, v \in e_i, i = 1..., l_v\}, l_v$ is the number of edges coincidenting with the vertex v

   calculate $\sum_i p(v_i), \{v, v_i\} = e_i$

   if $p(v) \le \sum_i p(v_i)$, then $V' = V' \cup \{v\}, d(v_i) = d(v_i) - 1$, else

   $V' = V' \cup \{v_i\}, d(v_i) = d(v_i) - 1$

   $V_3 = V_3 \cup \{v_i\}$

   $E_1 = E_1 - \{e_i\}, E_2 = E_2 - \{e_i\}$, i.e. the edge e_i is covered by the vertex v and then it is the degree of a neighbouring vertex v_i nears.

   if $v_i \in V_2$ then $V_2 = V_2 - \{v_i\}$

   if $|V_2|$>0, go to 2., else

3. $c = c + 1$, go to 1.

4. $r = r - 1$

   if $|E_1|$>0, go to 1., else

   V' is the set of the vertices, which we have been looking for. End.

3.2 Algorithm for vertex cover with the price of vertices: (segregation in the overall vertices after the first algorithm, 2-nd algorithm)

read V, V is the set of vertices of the graph.

read E, E is the set of edges of the graph.

$V_1 = V, E_1 = E$

find $r\min, v_{r\min}$ are the vertices of the maximal degree in V .

find $c_{\min}, c_{\max}$ are the maximal and minimal price of the vertices.

put $r = r\,min, c = c_{min}, V'' = \emptyset$ (V'' is a created locked optimal set of vertices)

read V', V' - is the set of vertices from the algorithm 1

1. put $V_2, V_2 = \{v, v \in V', d(v) = r, p(v) = c\}$
2. choose arbitrary $v \in V_2$

   $E_2 = \{e_i, \{v, v_i\} = e_i, v_i \in V', i = 1..., l_v\}$

   calculate $\sum_i p(v_i), \{v, v_i\} = e_i$

   if $p(v) \le \sum_i p(v_i)$, then $V'' = V'' \cup \{v\}, V_2 = V_2 - \{v\}, d(v_i) = d(v_i) - 1$, else

   $V'' = V'' \cup \{v_i\}, d(v_i) = d(v_i) - 1$

   $E_1 = E_1 - \{e_i\}, E_2 = E_2 - \{e_i\}$

if $v_i \in V_2, V_2 = V_2 - \{v_i\}$
if $|V_2|$>0, go to 2., else
3. $c = c + 1$, go to 1.
4. $r = r + 1$
if $|E_2|$>0, go to 1., else
V'' is the set of the vertices, which we have been looking for. End.

4 CONCLUSION

On the contrary of algorithm for Vertex Cover this algorithm deals with optimal set of vertices in a manner of different prices of vertices and chooses minimal price of vertices. The algorithm recognizes prices of vertices therefore the price of the vertex is compared with sum of prices of neighbouring vertices. A decision of which vertices will the minimal set consist of is based on this fact. In the practical part the author compares the efficiency of algorithm with the exact solution.

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