NONLINEAR MODEL OF THE VEHICLE HYDROPNEUMATIC SUSPENSION UNIT

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Summary: The lumped parameters model of the hydropneumatic suspension unit and its verification is described in the paper. The relations effective surface and volume of the rubber bellows air spring versus position and pressure and Bernoulli equation are used in the mathematical model of hydraulic part. The oil volume compression changes are considered negligible. The adiabatic state equation for the gas is used. The gas flow through the throttling injector is considered both in under and over-critical conditions. The static and dynamic characteristics of the whole unit and its parts were measured.

Key words: hydropneumatic suspension unit, rubber bellows air spring, mathematical model.

INTRODUCTION

The hydropneumatic suspension unit (fig. 1) consists of hydraulic and pneumatic parts. It is possible to separate the hydraulic part for the measurement (1, 2).

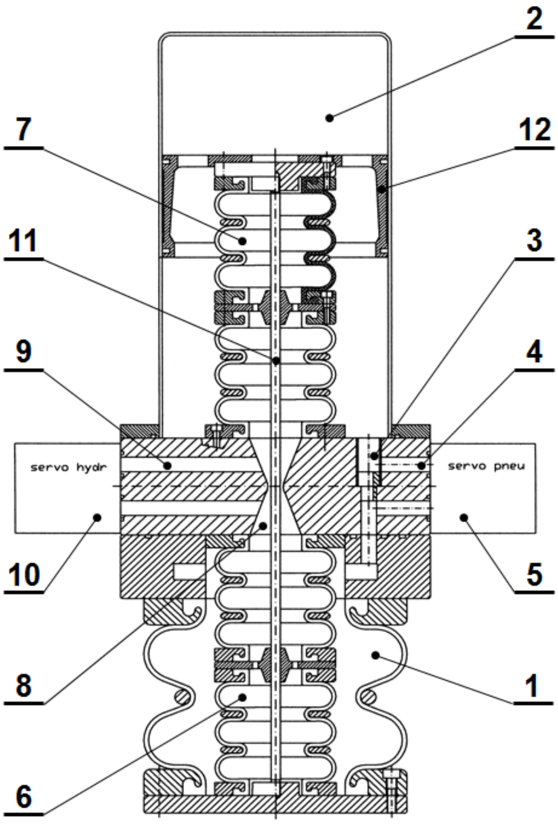

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Fig. 1 – Hydropneumatic suspension unit

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The unit (see fig. 1) consists of: pneumatic rubber bellows air spring (1), steel chamber (2), pneumatic actuator and channels (3, 4 and 5), lower and upper rubber bellows spring (6 and 7, silicon oil filled), fixed channel (8), hydraulic actuator and channels (9, 10), steel rod (11) and lateral guidance system of upper rubber bellows spring (12).

1. NONLINEAR MATHEMATICAL MODEL

1.1 Hydraulic part

The hydraulic part has the character of a classic damper, but it has small spring character as well. Hydraulic part consists of two rubber bellows springs, which are connected with each other through a throttling bore and parallel hydraulic channels with actuator.

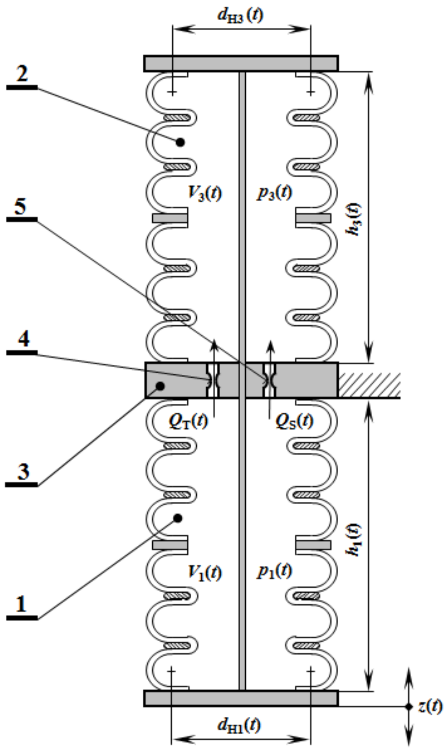

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Fig. 2 – Scheme of the hydraulic part of the unit (1, 2 – lower and upper rubber bellows springs, 3 – fixed point of measurement equipment, 4 – throttling bore, 5 – servo valve)

The relations between position and pressure of one rubber bellows spring and the effective surface were measured using laboratory purpose-built testing equipment. The relation volume versus position and pressure was approximated with function (1).

The volume approximation function was chosen in the following form

$$V(z,p) = a_{V0} + a_{V1}z + a_{V2}e^{a_{V3}z} + a_{V4}p + a_{V5}z\,p \quad , \qquad (1)$$

where V is volume, z is position, p is pressure, a_{V0} up to a_{V5} are coefficients.

The coefficients a_{V4} and a_{V5} were determined experimentally from the measurement on the assembled hydraulic part. The explicit expression $p = p(z, V)$ from (1) is

$$p = \frac{V - a_{V0} - a_{V1}\,z - a_{V2}\,e^{a_{V3}z}}{a_{V4} + a_{V5}\,z} \quad . \qquad (2)$$

The pressure inside e.g. lower rubber bellows spring as a function of time is described by

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\partial p}{\partial z}\frac{\mathrm{d}z}{\mathrm{d}t} + \frac{\partial p}{\partial V} Q_V \,. \qquad (3)$$

$Q_V = \mathrm{d}V/\mathrm{d}t$ is oil volume flow into the rubber bellows spring.

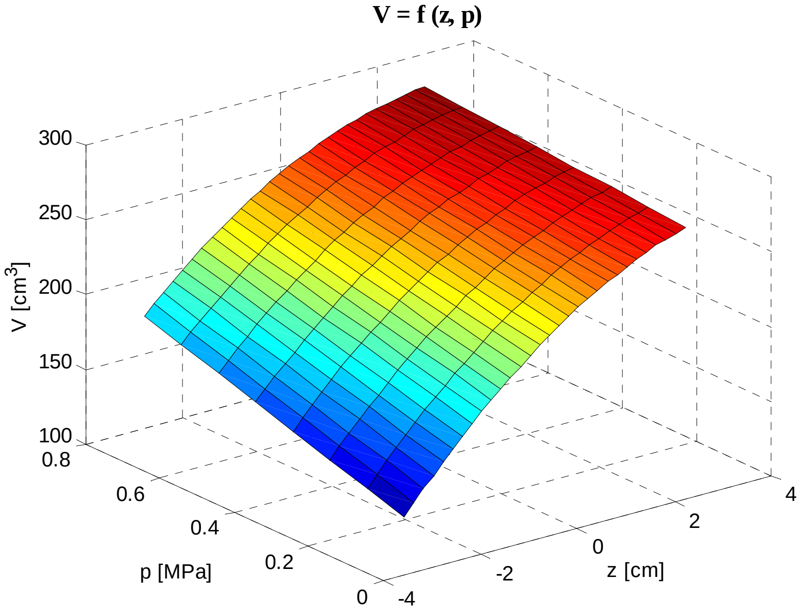

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Fig. 3 – 3D visualization of the function $V = V(z, p)$

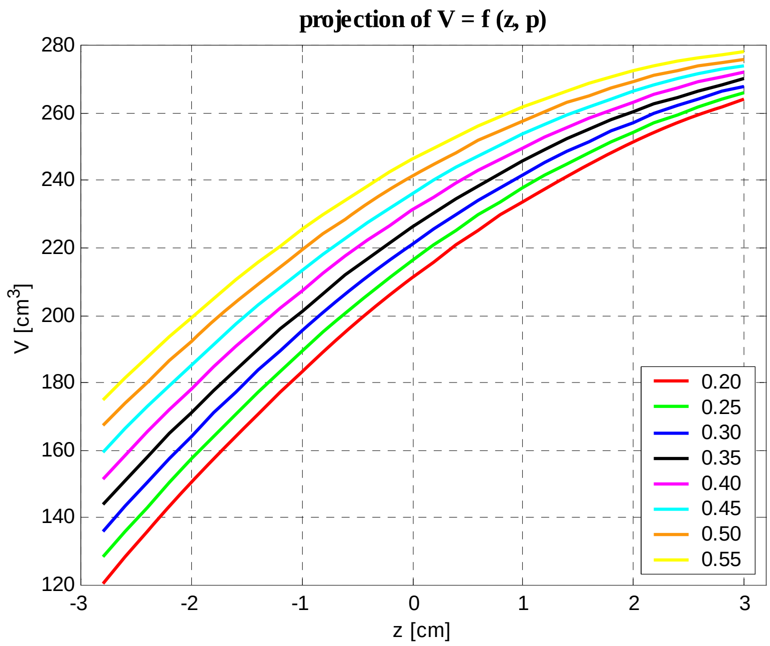

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Fig. 4 – Projection of cross section curves of the function $V = V(z, p)$, curves parameter is pressure $p = 0.20$ up to 0.55 MPa

Let us compute the derivatives $\partial p/\partial z$ and $\partial p/\partial V$ from (2) and substitute them into (3)

$$\frac{\mathrm{d}p}{\mathrm{d}t}=\frac{(-a_{V1}-a_{V2}a_{V3}e^{a_{V3}z})(a_{V4}+a_{V5}z)-(V-a_{V0}-a_{V1}z-a_{V2}ze^{a_{V3}z})a_{V5}}{(a_{V4}+a_{V5}z)^2}\frac{\mathrm{d}z}{\mathrm{d}t}+\frac{Q_V}{a_{V4}+a_{V5}z} \quad (4)$$

Let us designate the pressures inside the lower and the upper bellows and difference between them p_1, p_3 and $\Delta p = p_3 - p_1$ respectively. The following approximation of the solution of Bernoulli's equation was used to compute Q_V

$$Q_V=\mathrm{sign}(\Delta p)\ \frac{1-\mathrm{e}^{-\frac{|\Delta p|}{\tau_\mathrm{h}}}}{R_\mathrm{h}}\ , \quad (5)$$

R_h is a hydraulic resistance and τ_h is coefficient which affects the derivative $\mathrm{d}Q_V/\mathrm{d}(\Delta p)$ for $\Delta p = 0$.

The force

$$F_\mathrm{h}=F_3-F_1=S_{\mathrm{hef}3}p_3-S_{\mathrm{hef}1}p_1\ , \quad (6)$$

S_{hef1} and S_{hef3} are effective surfaces of the upper and lower bellows springs. The approximation of the effective surface of bellows spring was chosen in the following form

$$S_\mathrm{hef}(z,p)=a_{S0}+a_{S1}z+a_{S2}\,e^{a_{S3}z}+a_{S4}p+a_{S5}z\,p\,. \quad (7)$$

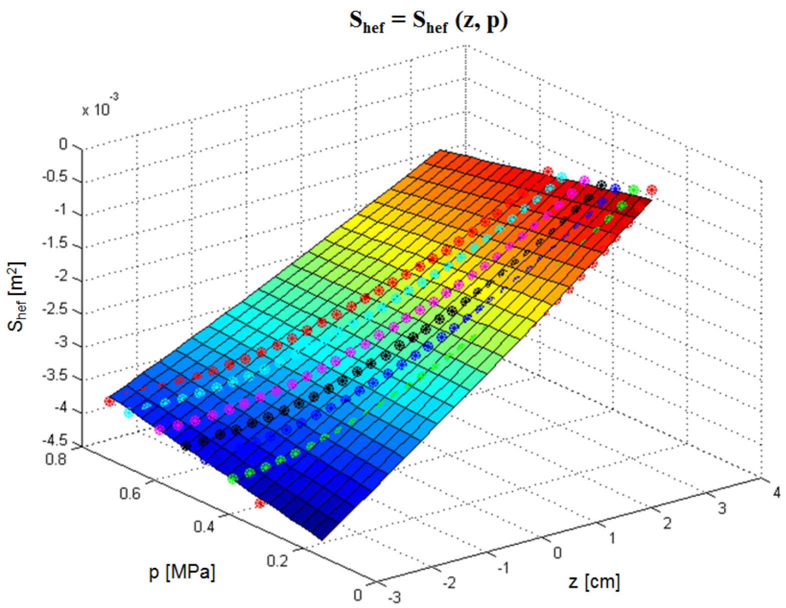

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Fig. 5 – 3D visualization of the approximation $S_\mathrm{hef} = S_\mathrm{hef}(z, p)$ and measurements of an effective surface

1.2 Pneumatic part

The pneumatic part of the suspension unit consists of rubber bellows springs, steel chamber and actuator – throttling element (see fig. 6).

The equation of adiabatic changes in air is

$$p\,\rho^{-\kappa}=konst\ , \quad (8)$$

where p is pressure, ρ is density and κ=1.4. The differential form of (8) is

$$\frac{\mathrm{d}p}{\mathrm{d}t}\rho^{-\kappa}-\kappa\, p\, \rho^{-\kappa-1}\frac{\mathrm{d}\rho}{\mathrm{d}t}=0\ . \tag{9}$$

From (9)

$$\frac{\mathrm{d}p_2}{\mathrm{d}t}=\kappa\,\frac{p_2}{\rho_2}\frac{\mathrm{d}\rho_2}{\mathrm{d}t} \tag{10}$$

and

$$\frac{\mathrm{d}p_4}{\mathrm{d}t}=\kappa\,\frac{p_4}{\rho_4}\frac{\mathrm{d}\rho_4}{\mathrm{d}t}\ . \tag{11}$$

Mass conservation law is

$$m=\rho\, V=konst \tag{12}$$

and the differential form of this law is

$$\frac{\mathrm{d}m}{\mathrm{d}t}=\frac{\mathrm{d}\rho}{\mathrm{d}t}V+\rho\frac{\mathrm{d}V}{\mathrm{d}t}=Q_{\mathrm{m}}\ , \tag{13}$$

Q_{m} is mass flow.

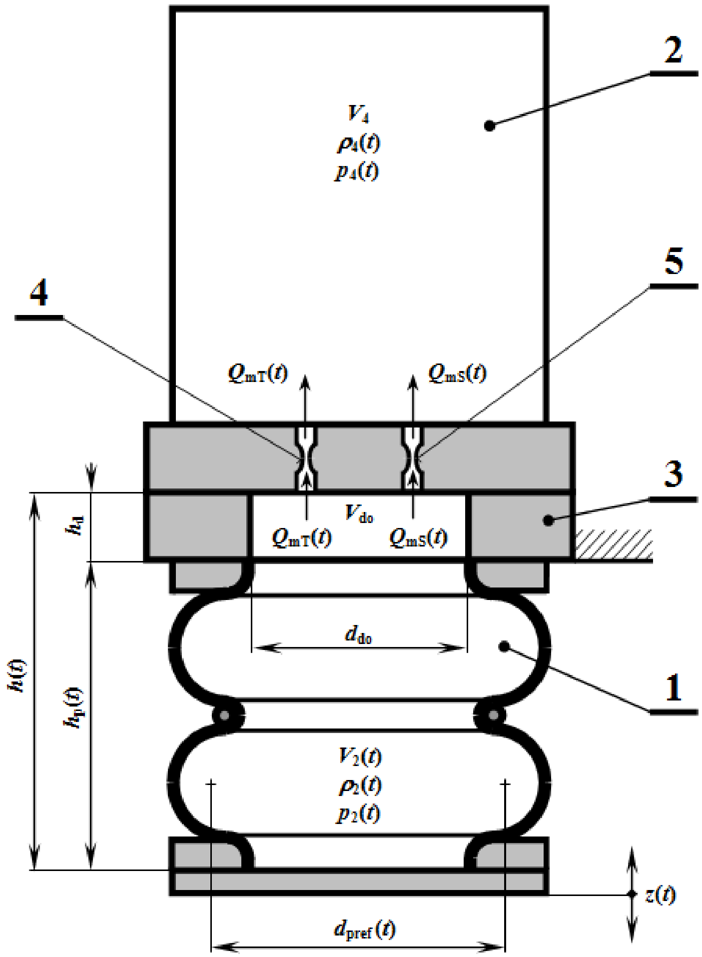

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Fig. 6 – Scheme of the pneumatic part of the unit (1 – bellows air spring, 2 – steel chamber, 3 – fixed point of measurement equipment, 4 – throttling injector, loss coefficient α_{PT}, 5 – servo valve, loss coefficient α_{PS})

The modifications of equation (13) for both parts of pneumatic unit are

$$\frac{\mathrm{d}\rho_2}{\mathrm{d}t}(V_2 - V_1) + \rho_2 \left[\frac{\mathrm{d}V_2}{\mathrm{d}t} - \frac{\mathrm{d}V_1}{\mathrm{d}t}\right] = -Q_\mathrm{m} \qquad (14)$$

and

$$\frac{\mathrm{d}\rho_4}{\mathrm{d}t}(V_4 - V_3) + \rho_4 \left[-\frac{\mathrm{d}V_3}{\mathrm{d}t}\right] = Q_\mathrm{m} \ . \qquad (15)$$

From (14) and (15)

$$\frac{\mathrm{d}\rho_2}{\mathrm{d}t} = \frac{1}{V_2 - V_1}\left\{Q_\mathrm{m} + \rho_2 \left[\frac{\mathrm{d}V_2}{\mathrm{d}t} - \frac{\mathrm{d}V_1}{\mathrm{d}t}\right]\right\} , \qquad (16)$$

respectively

$$\frac{\mathrm{d}\rho_4}{\mathrm{d}t} = \frac{1}{V_4 - V_3}\left\{Q_\mathrm{m} + \rho_4 \left[-\frac{\mathrm{d}V_3}{\mathrm{d}t}\right]\right\} . \qquad (17)$$

Bernoulli equation is used for the adiabatic gas flow through the throttling injector

$$\frac{v_{\mathrm{i}1}^2}{2} + \frac{\kappa}{\kappa - 1}\ \frac{p_{\mathrm{i}1}}{\rho_{\mathrm{i}1}} = \frac{v_{\mathrm{i}2}^2}{2} + \frac{\kappa}{\kappa - 1}\ \frac{p_{\mathrm{i}2}}{\rho_{\mathrm{i}2}} = konst \ , \qquad (18)$$

$p_{\mathrm{i}1}, v_{\mathrm{i}1}, \rho_{\mathrm{i}1}$ and $p_{\mathrm{i}2}, v_{\mathrm{i}2}, \rho_{\mathrm{i}2}$ are variables, which describe the flow in front of and behind the throttling injector. Since $v_{\mathrm{i}1} << v_{\mathrm{i}2}$, the relation between flow velocities is

$$v_{\mathrm{i}2} = \sqrt{\frac{2\kappa}{\kappa - 1}\ \frac{p_{\mathrm{i}1}}{\rho_{\mathrm{i}1}}\left[1 - \left(\frac{p_{\mathrm{i}2}}{p_{\mathrm{i}1}}\right)^{\frac{\kappa-1}{\kappa}}\right]} \ . \qquad (19)$$

For $p_{\mathrm{i}1}, \rho_{\mathrm{i}1}$ and $p_{\mathrm{i}2}, \rho_{\mathrm{i}2}$ from (8)

$$\rho_{\mathrm{i}2} = \left(\frac{p_{\mathrm{i}2}}{p_{\mathrm{i}1}}\right)^{\frac{1}{\kappa}} \rho_{\mathrm{i}1} \ . \qquad (20)$$

The mass flow through the throttling injector is

$$Q_\mathrm{m} = \rho_{\mathrm{i}2}\ S\ v_{\mathrm{i}2} \ , \qquad (21)$$

S is the sectional area of the throttling injector. After substitution of (19) and (20) into (21) and using the loss coefficient is

$$Q_\mathrm{m} = \alpha\ S\ \sqrt{p_{\mathrm{i}1}\rho_{\mathrm{i}1}}\ \sqrt{\frac{2\kappa}{\kappa - 1}\left[\left(\frac{p_{\mathrm{i}2}}{p_{\mathrm{i}1}}\right)^{\frac{2}{\kappa}} - \left(\frac{p_{\mathrm{i}2}}{p_{\mathrm{i}1}}\right)^{\frac{\kappa+1}{\kappa}}\right]} \ . \qquad (22)$$

This equation is valid for under-critical (laminar) flow, in the case of air it means for $p_{\mathrm{i}2}/p_{\mathrm{i}1} \geq 0.528$. Let us designate $p_\mathrm{krit}/p_{\mathrm{i}1} = 0.528$. For over-critical (turbulent) flow, for $p_{\mathrm{i}2}/p_{\mathrm{i}1} < 0.528$ is

$$Q_\mathrm{m} = \alpha\ S\ \sqrt{p_{\mathrm{i}1}\ \rho_{\mathrm{i}1}}\ \sqrt{\frac{2\kappa}{\kappa - 1}\left[\left(\frac{p_\mathrm{krit}}{p_{\mathrm{i}1}}\right)^{\frac{2}{\kappa}} - \left(\frac{p_\mathrm{krit}}{p_{\mathrm{i}1}}\right)^{\frac{\kappa+1}{\kappa}}\right]} \ . \qquad (23)$$

The resulting force is

$$F = F_1 + F_2 - F_3 - G \quad . \tag{23}$$

The force F_2 of the rubber bellows air spring (fig. 7) is

$$F_2 = S_{pef}\ p_2 \quad . \tag{24}$$

The dependences of effective surface S_{pef} and volume V_2 on position (see fig. 8) were approximated with fifth degrees polynomials.

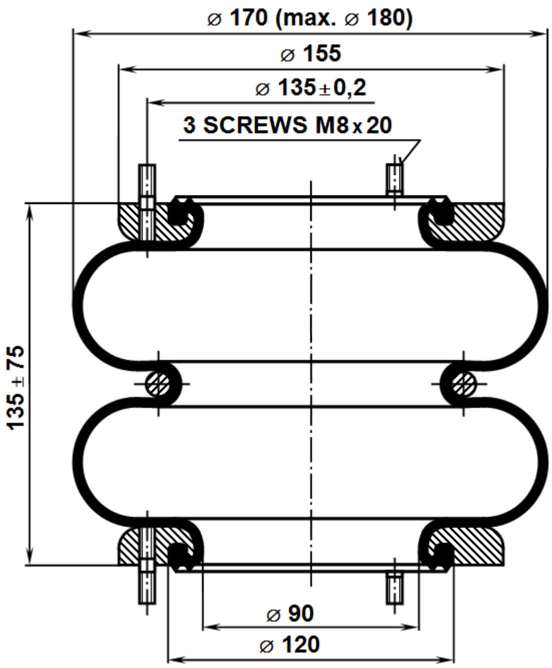

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Fig. 7 – Scheme of the rubber bellows air spring Rubena VD 135-07

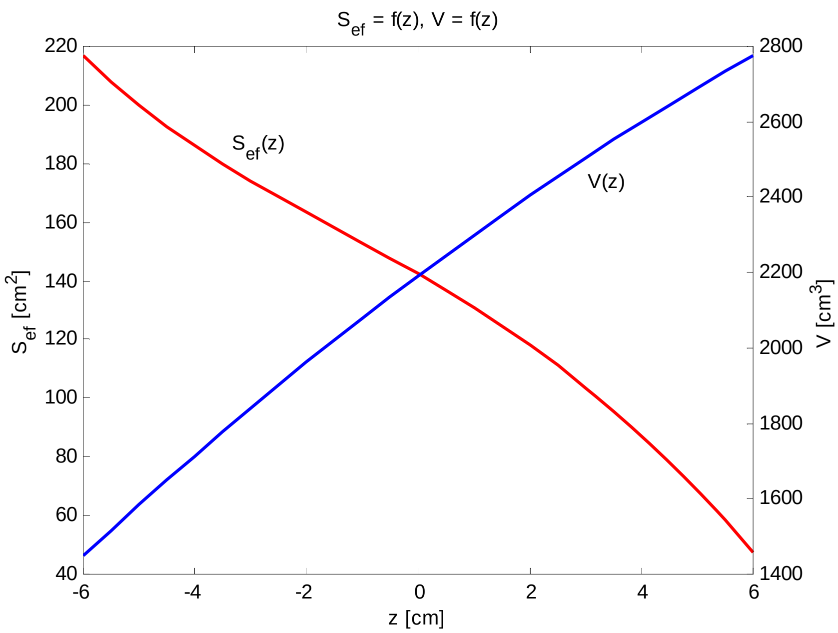

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Fig. 8 – Effective surface and volume of the used bellows spring Rubena VD 135-07 ($S_{pef} \equiv S_{ef}$ and $V_2 \equiv V$)

2. SIMULATION

The simulation model was created in MATLAB – Simulink. Plots of an axial force $F(z)$ acquired by numerical simulation are in figs. 9 up to 14, the frequencies of the harmonic signal were 0.416, 1.25 and 3.5 Hz, the amplitudes were 50, 50 and 30 mm.

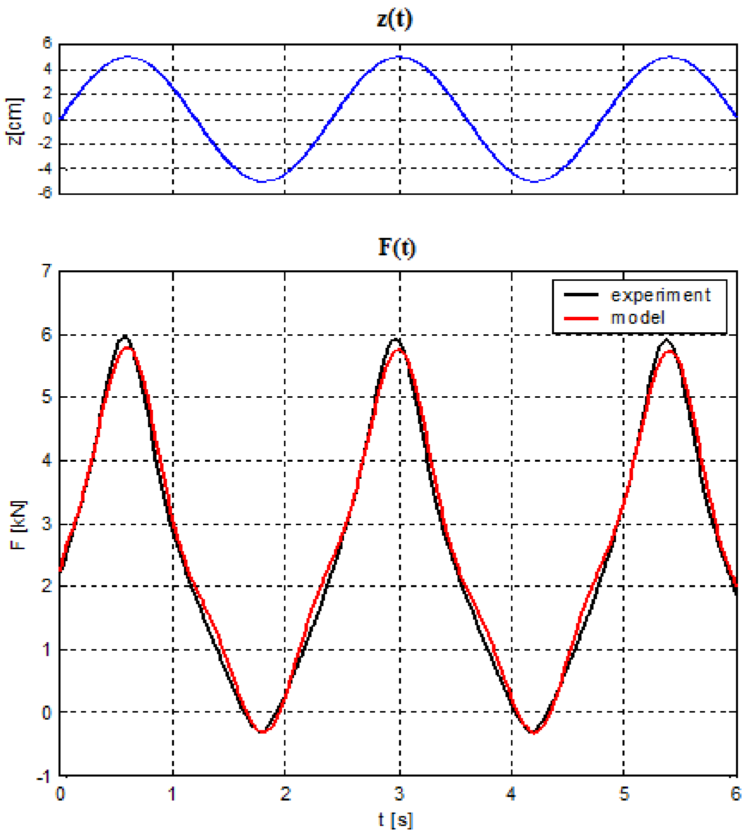

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Fig. 9 – Comparison of the model and experimental data for frequency 0.416 Hz and amplitude 50 mm (axial force $F = F(t)$)

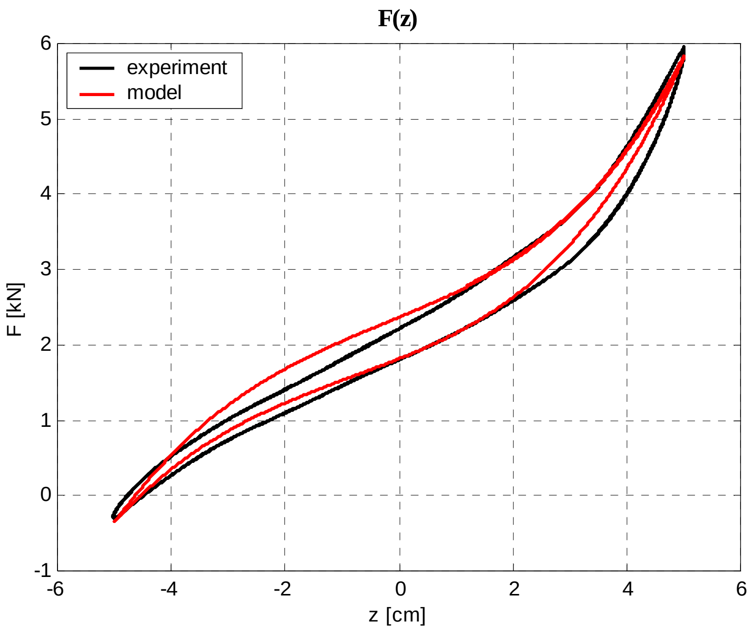

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Fig. 10 – Comparison of the model and experimental data for frequency 0.416 Hz and amplitude 50 mm (axial force $F = F(z)$)

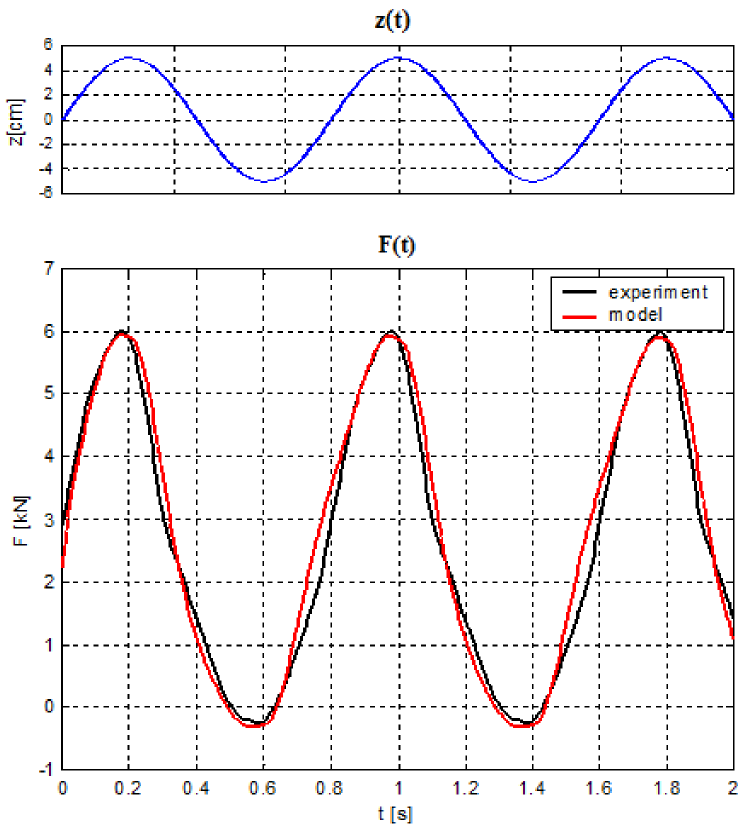

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Fig. 11 – Comparison of the model and experimental data for frequency 1.25 Hz and amplitude 50 mm (axial force $F = F(t)$)

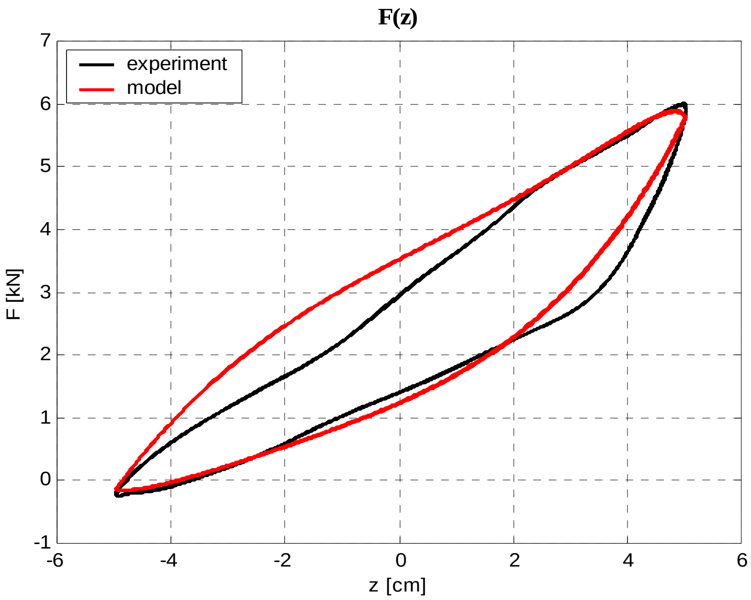

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Fig. 12 – Comparison of the model and experimental data for frequency 1.25 Hz and amplitude 50 mm (axial force $F = F(z)$)

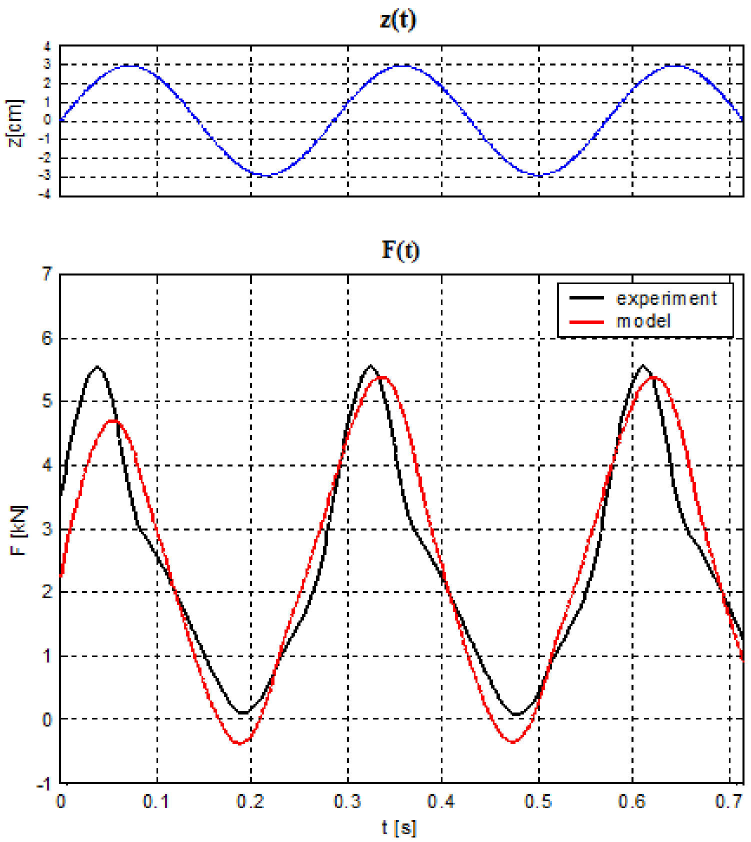

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Fig. 13 – Comparison of the model and experimental data for frequency 3.5 Hz and amplitude 30 mm (axial force $F = F(t)$)

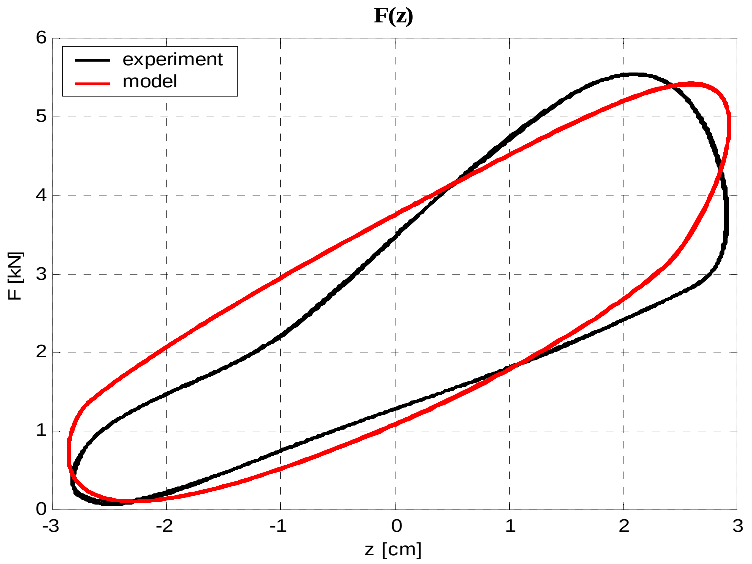

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Fig. 14 – Comparison of the model and experimental data for frequency 3.5 Hz and amplitude 30 mm (axial force $F = F(z)$)

CONSLUSIONS

The development of a hydropneumatic suspension unit (with rubber bellows springs) is described in the paper. The simulation model was created in MATLAB – Simulink. This model matches the dynamics of the hydropneumatic unit under consideration relatively well. Prospective application of this unit is truck back axle suspension. The spring and damping characteristics can be changed in the case of semi-active, or active control. The control has not been designed yet.

ACKNOWLEDGMENT

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