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# INFLUENCE OF EDGE REINFORCING RING ROTATION ON LOAD CARRYING CAPACITY OF CONICAL SHELLS LOADED BY EXTERNAL PRESSURE

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**Abstract** *The aim of the article is to present a part of the problem of calculating the load carrying capacity of conical shells, which by their dimensions and selected boundary conditions do not come under the scope of the standards. The investigated conical shells have a semi-vertex angle in the range of 75 ° - 85 ° and are provided with a reinforcing ring at the lower edge. When the conical shells are loaded by external pressure, not only does the edge circumferential ring move in the radial direction but also rotates (the moment has the direction of the tangent to the edge of the cone). This paper addresses the question of how this rotation affects the overall load carrying capacity of a conical shell. Since the investigated shell structures cannot be solved using standard methods and procedures, this problem is solved by means of numerical analyzes and experiments. On the basis of the results of numerical analyzes of the load carrying capacity of conical shells with different boundary conditions, the suitability of using simplified numerical models and samples for performing experiments was verified.*

**Keywords** *thin-walled structures, conical shells, FEM, load carrying capacity, loss of stability*

## 1 INTRODUCTION

Thin-walled steel shells are a common type of construction in many areas of industry, whether machinery, energy, or construction. They are used because of their relatively high load-carrying capacity while maintaining a low weight of the structure. Analytical methods are anchored in modern recommendations and standards for the design of such structures which solve the so-called standard structures (smooth cylindrical shell, conical shell, round plate, spherical shell, etc.). The scope of validity of calculation methods for a specific structure is precisely defined in the standards. This is especially the shape of the structure, the way it is supported and loaded. When designing structures that do not fall into the scope of standards (non-standard structures), it is necessary to proceed to numerical computational control or experiments. Computer programs based on the finite element method (FEM) are used for computational analyzes.

The assessment of the stability capacity of shell structures has been and is the subject of research by a number of scientists. The results of their work are enshrined in standards, regulations, and recommendations (European Recommendation for the Design of Steel Shell Structures ECCS, 2008; ČSN EN 1993-1-6, 2008; DIN 1880, 1990). One of the limit states that can occur when loading shell structures

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is the limit state of loss of stability. Then significant changes in geometry may occur and the carrying capacity of the structure is reduced at a stress level that may be well below the yield stress.

The loss of stability of a thin-walled shell structure can occur if excessive membrane compressive stress is caused in its wall due to the load. Equilibrium in pressure is redistributed to the equilibrium in pressure and bending (the so-called bifurcation for linear buckling of standard constructions) at the moment of buckling. Since the membrane stiffness of the shells is several orders of magnitude higher compared to the bending stiffness, the conversion of the membrane stress energy to the joint membrane and the bending stress energy is accompanied by visible deformations in the form of waves (shell buckling). At the same time, there is a significant reduction in the load carrying capacity of the shell, or complete collapse of the shell (see Fig. 1).

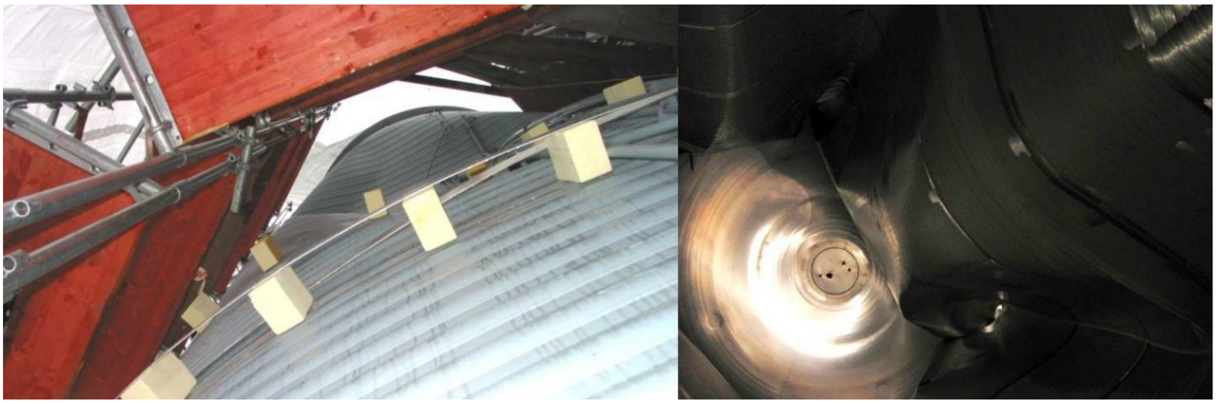

Fig. 1 Example of loss of stability of vertical storage tank (Paščenko, 2002)

Analytical formulas for solving linear stability of standard structures can be derived from basic differential equations (linear shell theory). Another possible method of solving the linear loss of stability is the LBA numerical FEM analysis (Esslinger, Van Impe, 1987). In these standard constructions, the membrane state predominates almost exclusively, which means that the analytical methods and theory of eigenvalues and eigenshapes can be easily applicable. In contrast, for non-standard shell structures such as the investigated conical shell with a large apex angle, there is a significant proportion of bending stress from the very beginning of the load and this shell then behaves very non-linearly under load (e.g., according to curve 0AC in Fig. 2; Bushnell, 1989).

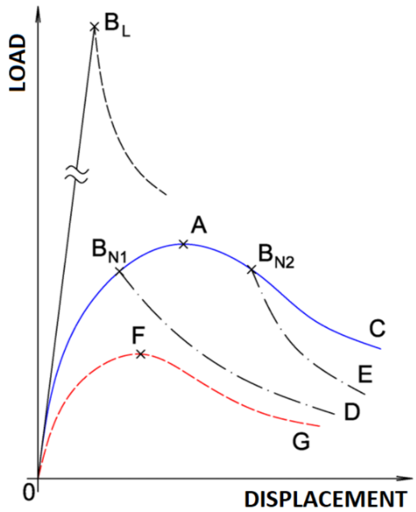


Fig. 2 Nonlinear response of nonstandard shell construction

Research of thin-walled shell structures that deviate from the validity of standards due to their geometry and boundary conditions structures has been conducted at the University of Pardubice. These are, among others, conical shells with a large apex angle and low-height spherical caps (Tomek 2012; Paščenko et al., 2013) and saddle supported cylindrical shells loaded by external pressure or axial compress force. The main goal of the research is to design pseudoanalytical methods for calculating the limit load of these shells, which use known formulas from standards for the calculation of standard structures. These formulas are supplemented by coefficients that take into account the considerable nonlinearity of the problem. In the thesis (Středová, 2012), conical shells with a semi-vertex angle of the range $\beta_c = 75° \div 85°$ were solved (see the scheme in Fig. 3). The solved boundary conditions include simply supported and hinged lower edge, which represent zero and infinitely large radial stiffness of the shell edge (Chryssanthopoulos, M., Spagnoli, A. 1997). Between these two extreme boundary conditions, there are conical shells with edge rings, which represent the final radial stiffness.

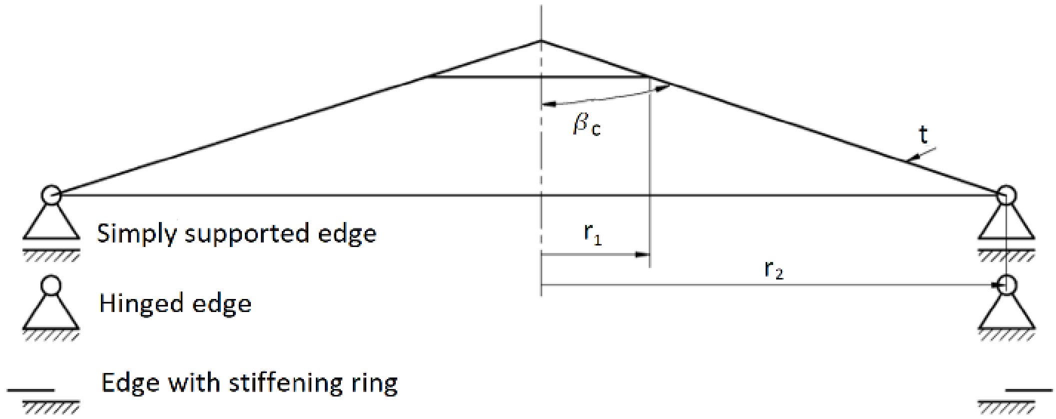


Fig. 3 Scheme of conical shell

The engineer cannot avoid a number of simplifying assumptions when creating a numerical model. Care must be taken to ensure that any deviation from the actual design is on the safe side. But excessive conservative assumptions can cause unnecessarily heavy weight and uneconomical construction. The model of the conical shell is relatively simple. However, it is necessary to ensure that the simplifications used do not significantly affect the results. This paper shows how the possible rotation of the edge ring affects the load-carrying capacity of the conical shell. The effect of rotation may prevail over the radial displacement (e.g., ČSN EN 13445-3, 2003 - dimensioning of arched bottoms. It is possible to assume that the influence of the radial displacement of the flexible rings will prevail over the influence of their rotation in the case of conical shells.

## 2 ROTATION OF THE REINFORCING RING

A typical use of thin-walled cones is the roof of cylindrical tanks. One of the objectives of this paper is to find boundary conditions for numerical and experimental models that can simulate a real connection of the cylindrical and conical shell. When the conical shells are loaded, not only does the edge ring move in the radial direction but also it can rotate. This chapter compares three models of conical shells that differ in the type of boundary conditions. It is necessary to verify which of the simpler conical shell models better matches the behavior of the real model of the conical roof.

### 2.1 Numerical model

Numerical analyzes are performed in the FEM program COSMOS/M. The models consist of quadrilateral 4-nodes shell elements SHELL4 (see Fig. 4). The radius of the lower edge of the conical shell is $r_2 = 150\ mm$, the thickness of the conical shell and the cylindrical shell (in the case of a conical shell with a part of the cylindrical shell) is $t = 0.8\ mm$, semi-vertex angle is $\beta_c = 80°$. The thickness of the ring with width $b = 15\ mm$ varies in the range $t_r = 0.4 - 20\ mm$ (i.e., the cross-sectional area of the ring is in the range $A_r = 6 - 300\ mm^2$). Numerical analyzes are of the GNA type (geometrically nonlinear analysis) type, which is an analysis of an ideal shell (i.e., without initial imperfections) with respect to geometric nonlinearity (large displacements), which includes changes in the shell geometry from the applied load. This type of analysis considers the elastic behavior of the material. The material is steel with mechanical properties $E = 210000\ MPa$ and $\mu = 0.3$. The numerical models are loaded by external pressure $p = 1\ MPa$. The Arc Length Control strategy is used to process control of nonlinear analysis.

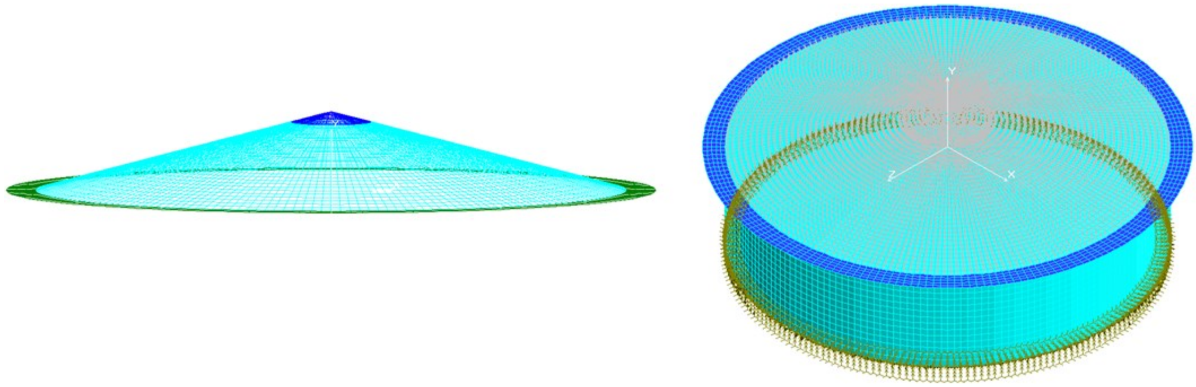

Fig. 4 Numerical models of conical shells with reinforcing rings

The models have the following boundary conditions:

- Simply supported conical shell with prevented ring rotation (model I).
- Simply supported conical shell with free ring rotation (model II).
- Conical shell model with part of the cylindrical shell, the lower edge of which is fixed (models III/A and III/B). These models represent, for example, a real cylindrical tank with a conical roof.

The evaluation of the deformed shape of the numerical model of the real conical roof (model III/B), after the limit state of loss of stability, follows. When loss of stability occurs, the conical shell deforms near the lower edge (see Fig. 5). The shape of the loss of stability of the conical shell (4 waves) affects the cylindrical shell of model III/B. There is a slight corrugation of the cylindrical shell in the axial and radial directions (total displacements of the cylindrical shell). This deformation could affect the load carrying capacity of the entire conical shell model with a cylindrical shell. It is necessary to analyze the influence of the transfer of deformations from the conical shell to the cylindrical shell on the carrying capacity to be able to compare the results of numerical analyzes with experiments. For this reason, an additional numerical model with an adjusted boundary condition is created (Model III/A). Vertical displacements of the nodes in the area of the connection between the cylindrical shell and the cone are prevented. This condition does not affect the rotation of the ring of the model with the cylindrical shell, but there is no corrugation of the cylindrical shell in the axial direction. However, this effect must be considered, e.g., by the appropriate reduction factor $C_r$. Thus, the investigated models of the conical shells (I and II) themselves do not accurately reflect reality.

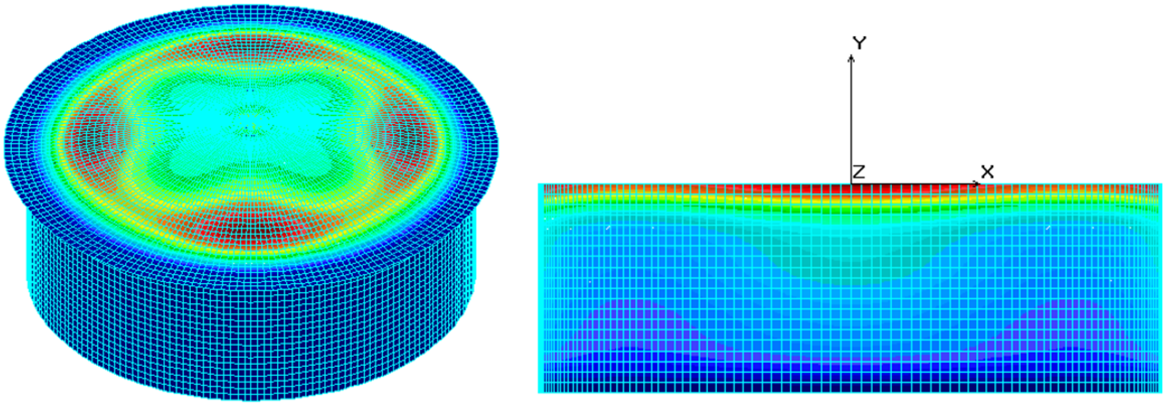


Fig. 5 The deformed shape of model III/B when loss of stability and corrugation of the edge of the cylindrical shell

## 2.2 Results of GNA

The limit external pressure values for individual numerical models are shown in the graph (Fig. 6) depending on the cross-sectional area of the ring and in Table 1. The graph also shows the dependence of the limit pressure of the cylindrical shell model without the additional boundary conditions mentioned above.

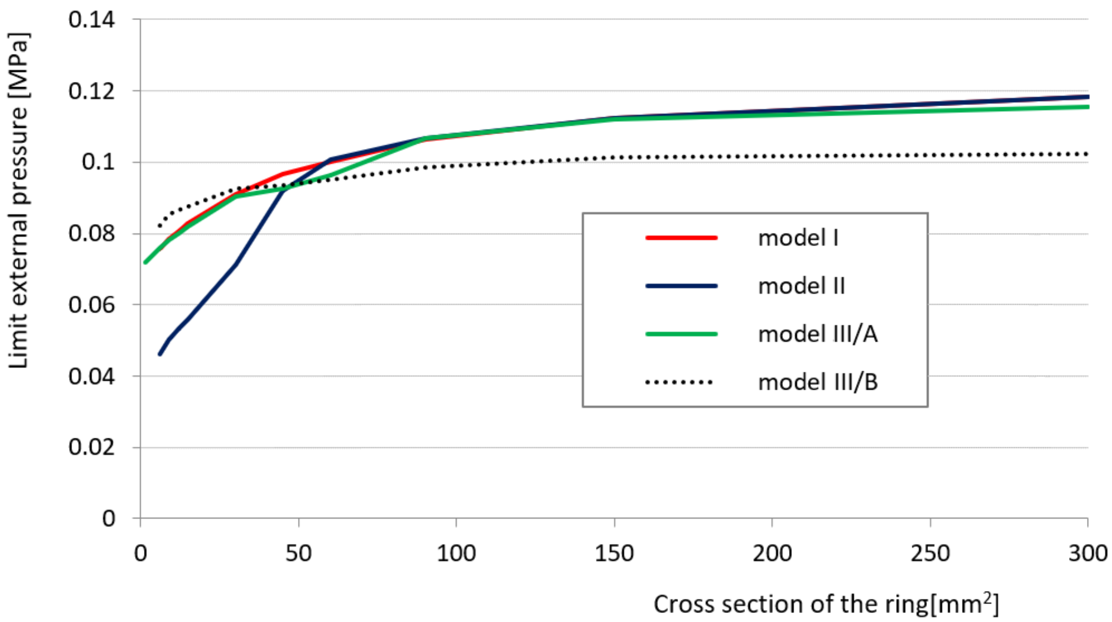


Fig. 6 Dependence of the limit external pressure of conical shell models on the ring cross-section area

Tab. 1 Results of GNA analyses

| ***Dimensions of conical shells*** | $t_r$[*mm*] | 0,4 | 0,6 | 0,8 | 1 | 2 | 3 | 4 | 6 | 10 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $A_r$[$mm^2$] | 6 | 9 | 12 | 15 | 30 | 45 | 60 | 90 | 150 | 300 |
| **Limit pressure $p_{GNA}$ [MPa]** | **Model I** | 0,07574 | 0,07836 | 0,08077 | 0,08292 | 0,09107 | 0,0967 | 0,10018 | 0,10624 | 0,11232 | 0,11841 |
| | **Model II** | 0,04608 | 0,05026 | 0,05336 | 0,0557 | 0,07112 | 0,09201 | 0,1008 | 0,10661 | 0,11237 | 0,11833 |
| | **Model III/B** | 0,07586 | 0,07818 | 0,08005 | 0,08195 | 0,09031 | 0,09266 | 0,09648 | 0,1067 | 0,1119 | 0,11547 |

The curves show that a more suitable replacement for the conical shell model with the attached cylindrical shell (III/A) is the conical shell model with a prevented rotation of the ring (model I). In this case, the model of the conical shell itself behaves very similarly to model III/A (the maximum error is 4.4 %). The model with the permitted rotation of the ring (II) approaches the values of the limit pressure of the model III/A only in the area of stiffer rings (approx. from $A_r = 60mm^2$).

As already mentioned, the load carrying capacity of model III without an additional boundary condition (i.e., III/B) is probably influenced by the transfer of deformation from the conical shell to the cylindrical shell. The maximum relative error of models I and III/A compared to model III/B is approximately 12 %. This error can be considered using a suitable reduction factor for example $C_r = 0.85$ after performing further series of numerical analyzes and verification experiments.

The model of the conical shell with the attached cylindrical shell (III) is replaced by the model of the conical shell with prevented rotation of the ring (I). In particular, this substitution achieves considerable simplification of verification experiments. It is necessary to prove whether this compensation is correct in further research. It will be necessary to perform a series of experiments with a model of a conical shell with part of a cylindrical shell. For experiments on the loss of stability of conical shells with a part of the cylindrical shell, it will be necessary to modify the test equipment. At present, the test device is designed for experiments on the loss of stability of samples of conical shells and spherical caps, which are simply placed on the device (samples with an edge ring are prevented from rotating the ring).

Figure 7 shows the edge details of the compared models. For the model III/A, the ring rotates.

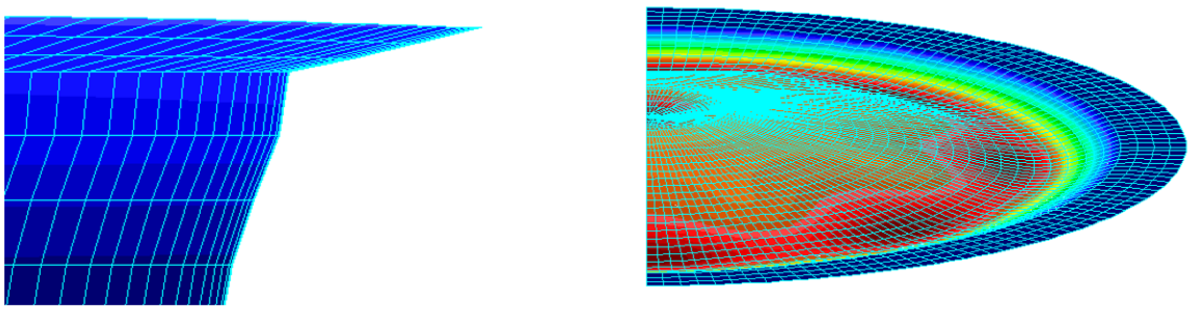

Fig. 7 Details of deformations of models III/A and I

## 2.3 Experiment

It is possible to simulate a number of experiments relatively quickly and cheaply when using numerical analyzes performed in computer programs based on the finite element method (FEM). In particular, the solution of complicated nonlinear problems requires the obtained results to be verified experimentally. Numerous simplifications occur when creating a numerical model. Therefore, it is not possible to trust unreservedly the results obtained numerically. It is possible to detect gross errors in the results by performing an experiment. This section presents a test device on which tests of loss of stability of conical shells and spherical caps (Paščenko, Tomek, 2011) are performed. Air is sucked out of the main cylinder (tube) by means of a pump, and thus an internal vacuum (external pressure) is created. The pressure value is read on the scale of the analog manometer until the limit value is reached, when the loss of stability of the test specimen occurs. Figure 8 shows a device with a test sample of a conical shell without an edge ring placed on a rubber seal.

Fig. 8 Vacuum test equipment for loss of stability tests

The test equipment was modified for testing shells with reinforcing rings. The results of the numerical analyzes presented in this article are used to adjust the holding of the samples on the test equipment. It is necessary to avoid rotation of the edge rings of the test specimens, but without limiting the radial displacement of the edge. The circumferential ring is placed on the O-ring and then clamped in the space between the retainer and the second flange (Fig. 9). The clearance is defined by bolts.

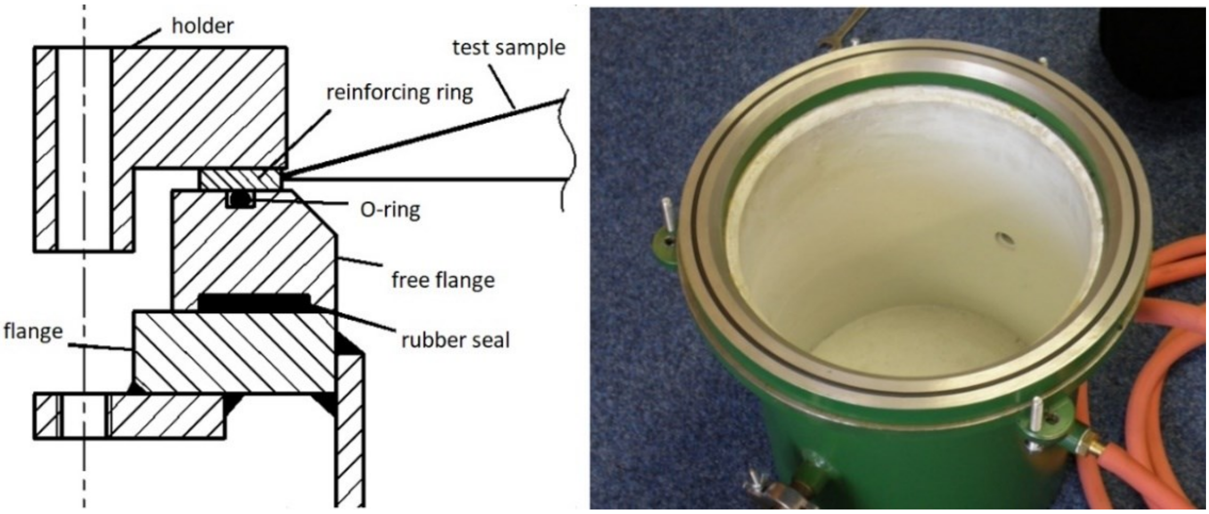


Fig. 9 Modification of test equipment for testing shells with edge rings

The results of the experiments are compared with the results of numerical analyzes of real shells of the GMNA type, where it is necessary to consider the nonlinear behavior of the material and further the presence of initial imperfections, due to the reduction coefficient $\alpha$ (ECCS, 2008; Poggi, 1997). The test specimen of a conical shell with an edge ring after an experiment is shown in Figure 10.

Table 2 shows the measured value of the limit pressure and the result of numerical analyzes of the GMNA type. The relatively large relative error of the results in this case is probably due to the presence of initial imperfections (mainly affected area of the meridian weld of the sample). The effect of initial imperfections was not considered in the numerical calculation. The influence of initial imperfections can be considered using the method specified in the standard (ECCS, 2008).

Fig. 10 Conical shell sample with reinforcing ring after the experiment

Tab. 2 Results of experiment and GMNA

| Sample number | Semi-vertex angle $\beta_c$ [°] | Cross section of the ring $A_r$ [$mm^2$] | Calculated limit external pressure $p_{GMNA}$ [MPa] | Measured limit external pressure $p_{exp}$ [MPa] |
|---|---|---|---|---|
| 10 | 80 | 30 | 0,08125 | 0,065 |

## 3 CONCLUSIONS

The aim of the article was to present the process of creating a numerical model and to verify the suitability of the simplifications of the numerical models used. In practice, the conical shell is reinforced with an edge ring and welded to the cylindrical shell. It was necessary to verify how this connection affects the load carrying capacity of the conical shell itself. It has been shown that the behavior of a conical shell with a cylindrical shell better captures the model of the shell with avoided rotation of the edge ring. These boundary conditions greatly simplified the numerical models used to determine the method for calculating the external overpressure limit and, last but not least, the production of test specimens.

The results presented in this paper led to a modification of the test equipment. Thanks to this modification, it is possible to test the test specimens of conical shells with an edge ring so that the ring can move in the radial direction during loading, but it is prevented from rotating.

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