COMPARISON OF VEHICLE HANDLING MODELS

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Summary: Vehicle handling quality is one of the most significant factor of the active road safety. This paper proposes and compares four variants of computational models usable for the vehicle handling analysis. The first three of them are based on single track model extended by Magic Formula tire model and lateral load transfer during cornering. The last one is a multi-body model created in MSC Admas software which is a three-dimensional model of the vehicle comprised of individual subsystems with many degrees of freedom. Presented models are validated with measurement of double lane change manoeuvre executed by the Formula Ford car.

Key words: vehicle handling, computational model, multi-body model, simulations

INTRODUCTION

Passenger car handling, mainly in critical situations, is one of the basic elements that influence the road safety. The vehicle handling can be evaluated in various ways but the most reliable method is measuring of vehicle's driving parameters during a particular manoeuvre. However, this approach is accompanied by many disadvantages, especially high costs for the implementation of the manoeuvres, time-consuming preparation of measurements, inseparability of the individual factors or complicated change of some important factors which influence the vehicle handling. All these drawbacks are eliminated by one method: simulation of manoeuvres with the support of a computational model of the vehicle. The greatest advantage of the simulations lies in the possibility of examining the vehicle handling already in the conception phase of the new car's design when the costs for changes in the construction are lowest. Moreover, the simulations can be used for sensitivity study of the impact of the basic automobile parameters on the vehicle handling. However, the more complicated the computational models are, the more difficult it is to ensure correct values of all vehicle parameters considered in the model, although the overall accuracy of the simulations usually improves. Computational models of the vehicle can be also used for the identification of the unknown characteristics of the automobile which cannot be measured directly – in such case the measurement on a real vehicle is linked with the simulations. One

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of the possibilities of the link between measuring and simulation is the so called Kalman filter (1). Its principle lies in the prediction of the output based on the entry data along with the correction based on the measured output. Even though the Kalman filter can also be utilized in computational models to analyse the handling, this paper does not discuss it. The aim of this work is to compare various mathematical models and to look for a certain compromise between the number of entry parameters and accuracy of the simulation computation. The simulation accuracy was evaluated based on the comparison of the computational results with the data measured in the real driving manoeuvre.

1. VEHICLE MODELS

This chapter describes and compares in detail individual variations of vehicle computational models.

1.1. Basic Single-track Model

Even if this is the simplest computational model for simulation of automobile driving manoeuvres, its validity has been verified in many studies (2, 3). As its name denotes, in this model the front and rear axle are reduced into one wheel since the slip angles of the wheels of one axle are usually identical. To evaluate the basic vehicle handling the longitudinal dynamics is not considered, therefore, the vehicle is moving with constant velocity during the manoeuvre. The dynamic load of tires caused by road roughness, road inclination or roll of the sprung mass is also disregarded. The scheme of the single-track model including the force effect is depicted in Fig. 1.

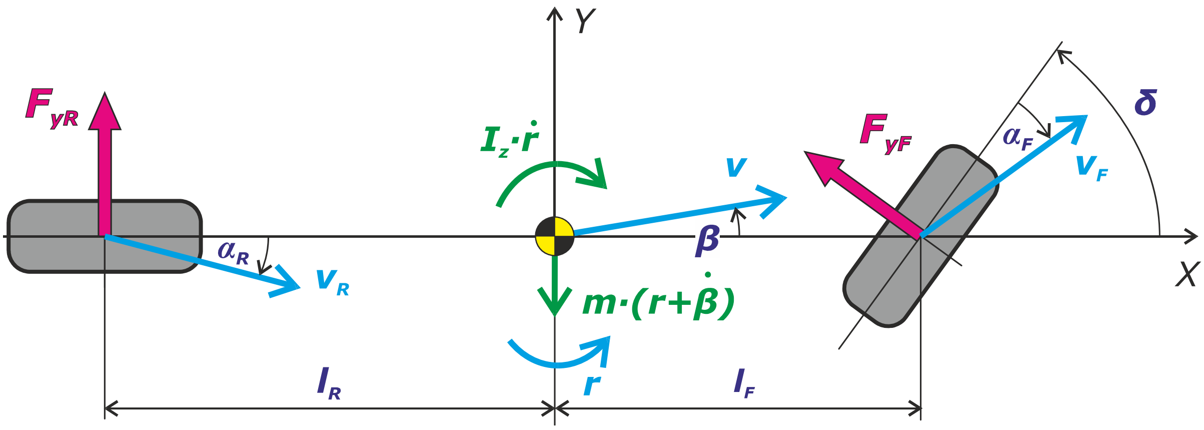

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Fig. 1 – Scheme of single track model of vehicle

In general, the model is based on a planar motion of a material point located in the centre of gravity of the vehicle with two degrees of freedom (2DOF) described by equations:

$$m \cdot v \cdot (r + \dot{\beta}) = F_{yF} \cdot \cos(\delta) + F_{yR} \tag{1}$$

$$I_z \cdot \dot{r} = l_F \cdot F_{yF} \cdot \cos(\delta) - l_R \cdot F_{yR} \tag{2}$$

Where:

F_{yF}	lateral force of front axle
F_{yR}	lateral force of rear axle
I_z	yaw moment of inertia
l_F	distance from front axle to centre of gravity
l_R	distance from rear axle to centre of gravity
m	vehicle mass
r	vehicle yaw rate
v	vehicle velocity
β	vehicle slip angle
δ	steering angle of front axle

The motion of vehicle model is controlled only by the velocity and the steering angle rate, while the vehicle handling is subsequently evaluated based on the results of the basic state parameters – yaw rate r and lateral acceleration a_y which is determined according to the following relation:

$$a_y = v \cdot \left(r + \dot{\beta}\right) \tag{3}$$

The lateral forces on the front and the rear axle, created by the elastic deformation of the tires, depend on the slip angle α and the cornering stiffness C_α, which is a primary tire characteristics expressing the linear relationship between generated lateral force and the slip angle.

$$F_{yF,R} = C_{\alpha F,R} \cdot \alpha_{F,R} \tag{4}$$

This approximation is valid only when small values of the slip angle are developed, i.e. in the linear stage of the tire behaviour (Fig. 2), therefore this model is not applicable for examination of the limit handling. The slip angles on the front and rear axle are determined as follows:

$$\alpha_F = \arctan\left(\frac{v \cdot \beta + l_F \cdot r}{v}\right) - \delta \tag{5}$$

$$\alpha_R = \arctan\left(\frac{v \cdot \beta - l_R \cdot r}{v}\right) \tag{6}$$

Overall, for the vehicle handling simulation with the use of single-track model only six input parameters characterizing the automobile are needed, specifically: mass, yaw moment of inertia, cornering stiffness of the front and the rear axle, and distance of the centre of gravity from the front and the rear axle. Of these parameters, the determination of the moment of inertia and cornering stiffness are the most complicated. If the vehicle is in the concept phase constructed in the CAD software, then it is possible to estimate the value of the moment of inertia from this virtual model. In the case of the simulation of a real vehicle manoeuvre, the most exact method of measuring the moment of inertia is with the use of the physical pendulum method (4). The tire cornering stiffness is obtained by measuring on a special test rig, or inversely by the use of single-track model based on a driving manoeuvre measurement (5). The low number of the vehicle entry parameters is a main advantage of the basic single-track model since measurement of many vehicle input parameters is quite complicated and therefore financially demanding. Due to the low number of entry parameters it is relatively simple to use this computational model in early development phases of the automobile, when not all characteristics of the vehicle have yet been set. The main disadvantages lie in the utilization of the model only in linear areas of the tire behaviour and in expected lower accuracy of the simulation results.

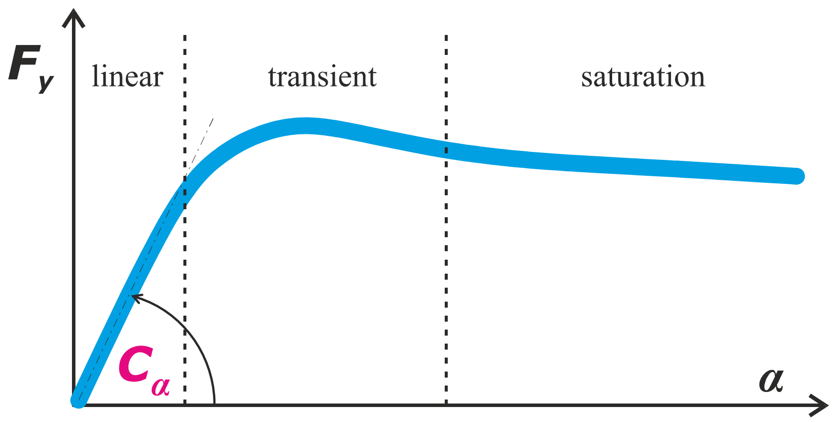

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Fig. 2 – Stages of tire lateral force – slip angle relation

1.2. Single-track Model + MF

This variant differs from the basic single-track model only in the tire characteristics. A nonlinear empirical model called Magic Formula (MF) is used to define the lateral forces on both axles (6). Fig. 2 illustrates a typical relation between the tire lateral force and slip angle which can be divided into three stages: linear, transient, and saturation. The lateral force is determined as follows:

$$F_y(\alpha) = D \cdot \sin[C \cdot \arctan(B \cdot \alpha - E \cdot (B \cdot \alpha - \arctan(B \cdot \alpha)))] \qquad (7)$$

The significance of the coefficients, which values are determined empirically, is as follows: B predominantly determines the curve gradient in the linear stage, C represents mainly the shape of the transient stage and partially the saturation stage, D specifies the maximal value of the lateral force, and E controls the shape of the saturation stage. Values of all these coefficients are not constant, but they depend on many other factors, e.g. wheel load, tire pressure, wheel camber, tire temperature, road surface, etc. However, in this single-track model no change in the wheel camber and load occurs, and all the other factors are considered constant, although it is necessary to determine the values of coefficients for given static load of front and rear wheels. The coefficients are usually determined for single tire, therefore the lateral forces on both axles are a double of one tire for given slip angle. The cornering stiffness in the basic single-track model approximates tire behaviour only in the linear stage, yet MF describes tire behaviour in all stages. Therefore, this model may be also used for the examination of the vehicle handling on the limit when slip angle values are in transient or saturation stage of the tire characteristics. The main disadvantage of the MF model may be seen in the identification of the tire characteristics coefficients. Nevertheless, these can be acquired, apart from measurements, from the tire manufacturers in some cases.

1.3. Single-track Model + MF + Load Transfer

This version of the computational model includes another important factor which influences the vehicle handling, lateral load transfer (LLT) on the wheels of an axle. LLT results mainly from the roll of the vehicle sprung mass and from radial reaction to the lateral acceleration. However, this computational model considers the vehicle as a single material point without differentiation between sprung and unsprung mass, therefore without the roll motion of the bodywork and the wheel camber change. The only difference against the previous version is determination of the load and the lateral force on each wheel of the axle. Therefore, this version consist of a single-track model with two degrees of freedom enhanced by the dynamic load of all four wheels. The quantity of the transferred load on both axles usually depends linearly on the lateral acceleration of the vehicle and on the parameter called axle roll stiffness K_R, which determines the amount of transferred load:

$$\Delta F_{zF,R} = K_{RF,R} \cdot a_y \tag{8}$$

The value of the roll stiffness of the front and the rear axle can be determined by calculation, or by steady-state manoeuvres of the vehicle (7). The load on the individual vehicle wheels is as follows:

$$F_{zFL,RL} = \frac{m \cdot g}{2} \cdot \frac{l_{R,F}}{l_F + l_R} - \Delta F_{zF,R} \tag{9}$$

$$F_{zFR,RR} = \frac{m \cdot g}{2} \cdot \frac{l_{R,F}}{l_F + l_R} + \Delta F_{zF,R} \tag{10}$$

As already mentioned above, the wheel load considerably influences the lateral force-slip angle relation and therefore the vehicle handling is also affected. The MF model of the tire (Pacejka '94 variant) considers this effect as the dependence of the individual coefficients (*B, D, E*) on other coefficients (a_1, a_2, a_3, a_4, a_6, a_7, a_{17}) which control the tire characteristic shape in respect to the wheel load. It means that it is necessary to measure the tire characteristics in required load range. Values of all coefficients are then determined from the results of the tire measurement. In this model modification the horizontal and vertical shift of the tire characteristics caused by the change in the wheel load are not considered.

$$D(F_z) = F_z \cdot (a_1 \cdot F_z + a_2) \tag{11}$$

$$B(F_z) = \frac{a_3 \cdot \sin\left(\arctan\left(\frac{F_z}{a_4}\right) \cdot 2\right)}{C \cdot D(F_z)} \tag{12}$$

$$E(F_z, \alpha) = (a_6 \cdot F_z + a_7) \cdot (1 - a_{17} \cdot \mathrm{sgn}(\alpha)) \tag{13}$$

The total lateral force on the axle is determined as a sum of lateral forces on the left and the right wheel. Similarly, as in the previous case of incorporating the nonlinear area of tire behaviour into the computational model, the main advantage of this model is the implementation of another factor, which should improve the correlation of simulations with the actual vehicle behaviour. However, it is at the expense of increased number of entry parameters for the tire model and also the computational model of the vehicle, which in the end means more demanding identification of values of the above mentioned factors.

1.4. Multi-body Model

The most sophisticated method for the vehicle driving manoeuvre simulations represents the multi-body dynamic model. The principle of this method is based on the dynamic, kinematic, and static analysis of the virtual mechanism, in our case an automobile. Basically, it is a three-dimensional model of the vehicle comprised of individual subsystems (parts), or system of bodies (multi-body) with many degrees of freedom. Multi-body software have been successfully used in many studies dealing with the vehicle handling (8, 9). To create a multi-body model the software MSC Adams was employed, specifically a module called Adams/Car which includes interactive graphical environment – with which a desired model of the vehicle may be easily created including the setting of the individual factors which affect the vehicle handling. Fig.3 illustrates a model of a vehicle created in the graphical environment of Adams/Car software. In the assembled model following factors were included: kinematics of wheel suspension of the front and the rear axle, wheel alignment, dampers and springs, steering mechanism and all mass characteristics, including the moment of inertia tensor. The MF model (2002 variant) was used again, including tire self-aligning torque, wheel camber and dynamic load of tire.

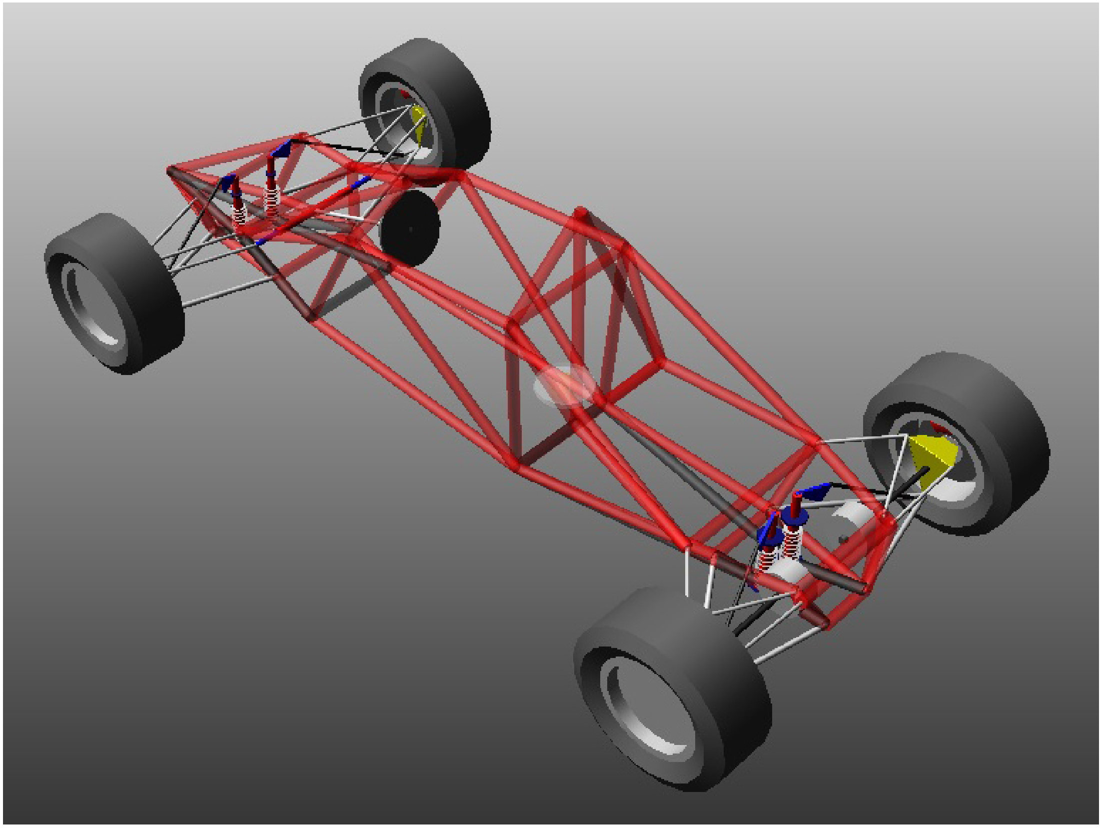

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Fig. 3 – Multi-body model of vehicle

Since a motion of the multi-body model with constant velocity was considered, all factors affecting the longitudinal dynamics of the vehicle (e.g. tire rolling resistance, traction system, longitudinal forces of tires, etc.) were ignored. The model does not present friction in the joints of suspension or elastic deformation of bodies, only rigid bodies were considered. The multi-body model motion is controlled by the velocity and the steering-wheel angle, similarly to the above mentioned single-track models which do not have the steering mechanism model. The relative position of the vehicle individual systems and the steering ratio between steering wheel angle and front wheel angle were assessed with the help of the 3D optical scanners; damper characteristics, and springs were measured on a shock dynamometer. The results of the multi-body simulations are the time responses of vehicle ride state parameters. The greatest advantage of the multi-body model is in the inclusion of immense number of parameters which influence the lateral dynamics and in the possibility to study their influence on the vehicle handling. However, to obtain credible results it is necessary to provide as precise values of all inputs as possible. Nevertheless, this could be very financially and time consuming when there is a demand to enter hundreds or even thousands values of parameters. However, the values of some of the parameters are predetermined or estimated in the early development of the vehicle, therefore acquiring the inputs becomes less demanding. Nonetheless, the simulations of driving maneuvers on already existing vehicle are more difficult because it is necessary to measure all parameters conscientiously.

2. MEASUREMENT

Measurements of driving manoeuvres were carried out on a real vehicle to validate all variants of the computational models and to determine their accuracy. The experiment was carried out at the Brno-Tuřany airport. Formula Faster Ford 1600 (Fig. 4) was used as an experimental vehicle to gather necessary data. The vehicle was equipped by a number of sensors to measure several state parameters. The capacitive accelerometer was employed to measure the lateral acceleration, the vibration gyroscope was used to determine the yaw rate, linear potentiometer situated on the steering rack was used to calculate the steering angle of front wheels and the optical sensor allowed to measure the velocity data of the vehicle with high precision. Data recordings of the steering angle and the vehicle velocity subsequently served as inputs for all computational models, while lateral acceleration, together with the vehicle yaw rate were used as references for the simulations accuracy assessment.

Source: Authors

Fig. 4 – Experimental vehicle – Formula Faster Ford 1600

The double lane change manoeuvre (also known as the moose test) was chosen for comparison of the models. This test is exactly defined by the ISO 3888:2011 norm (10), which delineates the required trajectory of the vehicle (Fig. 5), but also other conditions for the execution of the test – e.g. maintaining the constant speed of the vehicle during the manoeuvre, size and position of cones used for the marking of the track, etc. This manoeuvre is commonly used for the assessment of the vehicle handling in the linear area of behaviour as well as for the evaluation of critical situations when it is necessary to avoid an obstacle on the road without the loss of control over the vehicle. The manoeuvre is usually repeated with gradually increasing velocities until the maximum velocity with which the vehicle is still able to adhere to the track limits was reached. However, the main aim of this study was not to determine the limits of the formula car but to measure the reference data to assess the simulation models. The test was carried out several times with the constant speed of 65 km/h. Two runs which best fulfil required test conditions were chosen from the recorded data.

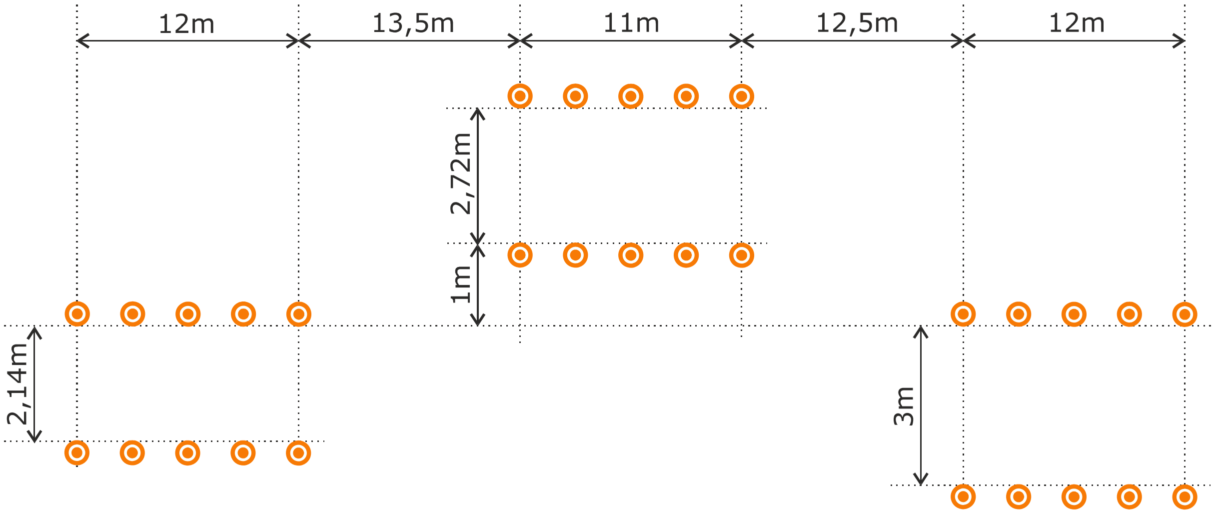

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Fig. 5 – Double lane change manoeuvre

3. COMPARISON OF THE VEHICLE MODELS

This part evaluates individual variants of the computational models of the vehicle based on the comparison of the simulation results with the data obtained from the above mentioned experiment. As a criterion for the results evaluation the average deviation between the values obtained from measurement x_{mer} and simulation x_{sim} relating to the maximal absolute value of the given quantity was chosen:

$$\varepsilon(x) = \frac{\frac{1}{n}\sum_{i=1}^{n}\left|x_{sim}(t_i) - x_{mer}(t_i)\right|}{\max\left|x_{mer}(t)\right|} \tag{14}$$

This criterion, called the normalized error of simulations, which was utilized to evaluate the accuracy of the computational models also in other studies (11), was determined for both most important output quantities, i.e. yaw rate $\varepsilon(r)$ and lateral acceleration $\varepsilon(a_y)$. Resulting values are shown in Tab. 1, comparison of the simulations and measurements for both chosen runs is depicted in Fig. 6 and 7.

Tab. 1 – Validation results: Normalized errors of yaw rate and lateral acceleration

Model variant	$\varepsilon(a_y)$			$\varepsilon(r)$		
	Run 1	Run 2	**Average**	Run 1	Run 2	**Average**
STM	10.6%	7.8%	**9.2%**	9.9%	7.6%	**8.8%**
STM+MF	11.9%	10.3%	**11.1%**	11.1%	8.2%	**9.7%**
STM+MF+LLT	11.3%	10.4%	**10.9%**	9.6%	8.2%	**8.9%**
ADAMS	7.0%	6.5%	**6.8%**	7.0%	7.5%	**7.3%**

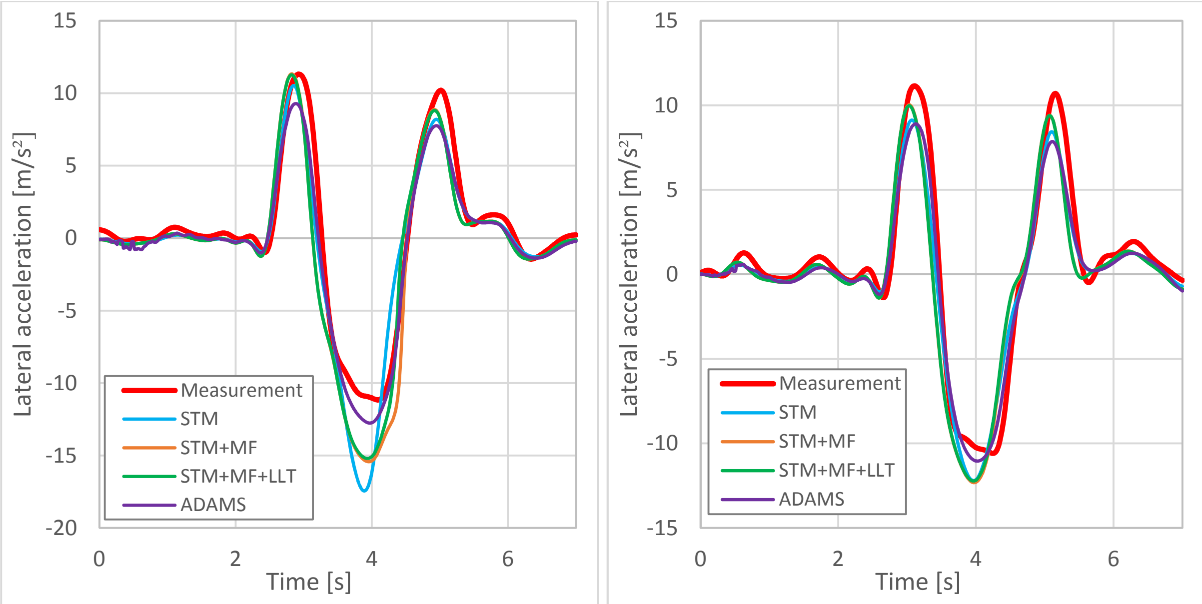

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Fig. 6 – Lateral acceleration for both runs

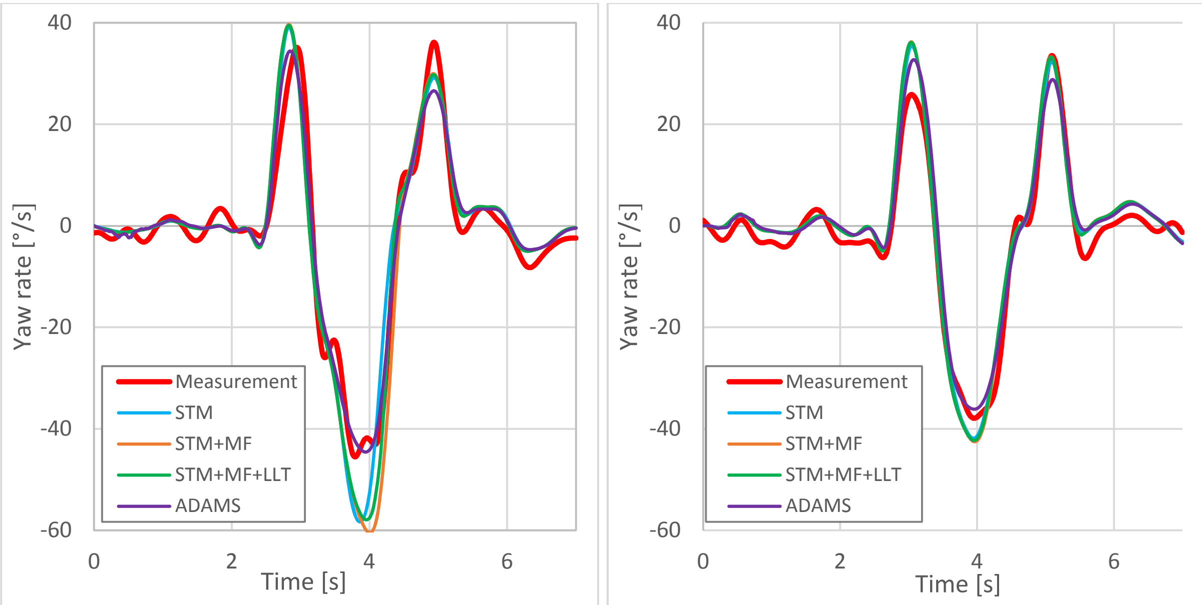

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Fig. 7 – Yaw rate for both runs

Since the trajectory of both runs was identical when the behaviour was compared, an evident difference in maximal values of yaw rate was detected. This is because of the divergent progress of the steering-wheel angle when the driving style was more aggressive (first run) – i.e. higher speed of steering-wheel especially in the first phase of the manoeuvre which demonstrates itself on the higher values of the yaw rate. However, the maximal values of the lateral acceleration in the individual phases of the manoeuvre are almost identical in both runs.

The multi-body model created in the ADAMS software comes closest to the actual measurement, not only in the shape of the characteristics, but also in the maximal values of the state parameters – this is also evident from the normalized errors of the simulations, which are the lowest from all models. The behaviour of all single-track models does not differentiate significantly from one another, the most important distinction is created during the first run in the phase when the vehicle is returned into the original trajectory – the models with Magic Formula reach the limit of tire lateral forces and therefore the delay in the lateral acceleration and yaw rate appears. This effect is most visible in the model without the lateral load transfer (LLT). All single-track models reach, in general, higher maximal values of both monitored parameters compared to the multi-body model. The most probable cause of these differences is that ADAMS model is a double-track model with a steering mechanism, which creates certain deviations of the front wheel angles and therefore also the response of the vehicle is different. When the normalized errors of individual single-track models were compared, the highest accuracy is reached with the basic single-track model. However, this may be caused mainly by the fact that the tires were operating only in the linear area of their behaviour during the measurement. If the critical behaviour of the vehicle was measured, the accuracy of the basic single-track model would be probably lower and the influence of the nonlinear model of the tire (MF) and the lateral load transfer would appear more markedly. Even if the fundamental presumption was confirmed – that the most complex model will be the most accurate one – another premise was not, i.e. that the accuracy of the simulation models increases with higher number of factors which influence the vehicle behaviour. This conclusion also corresponds with the results of other studies (12) – especially in the second run the differences between individual models are relatively small and all variants, at least with their curve shapes, match the measurement, even though the normalized errors of lateral acceleration differ only slightly. Nevertheless, all presented computational models, with higher or lower accuracy, may be used for simulation of driving manoeuvres. This insignificant influence of the examined factors of the single-track model would have to be verified on other vehicles or on different manoeuvres.

CONCLUSION

The most accurate of the examined computational models is the multi-body model created in ADAMS software. No dependency between the scope of the computational model and the results accuracy was detected. Every of the described models may be with certain accuracy used for examining the vehicle handling, since the detected differences in accuracy are not too high. The basic single-track model is especially suitable for the evaluation of the handling in the initial phases of the development of a new vehicle, when not all the parameters of the vehicle have been set yet. The complex multi-body model is then suitable for the detail examination of the handling in the final customization of the vehicle. These conclusions are valid only for the vehicle handling in the linear area of the tire behaviour, to assess the utilization of these simulations for handling in critical situations further research is needed.

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