EVALUATION OF VEHICLE HANDLING BY A SIMPLIFIED SINGLE TRACK MODEL

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Summary: This paper presents a simplified simulation method for investigation of vehicle handling behaviour using the single track model of automobile. The main aim of this approach is to create a simulation model which contains only very small number of parameters and still maintains sufficient accuracy. The procedures for determining the values of all the necessary parameters are briefly discussed, as is the validation process of the mathematical model against the measurements conducted during drive manoeuvres. On the basis of primary validation results, the model was slightly enhanced by the addition of new parameters.

Key words: vehicle, mathematical model, handling, simulations

INTRODUCTION

The handling of motor vehicles is one of the most important factors in road traffic safety. There are various methods for analysing handling performance, such as subjective evaluation of vehicle behaviour or objective measurements during a driving manoeuvre. Both methods are very effective for proving an existing automobile, but at the same time they are clearly not suitable in the early design phase of vehicle development, when there is no actual automobile to run. Also, the earlier the potential safety issues are discovered, the lower the funds that are subsequently needed to rectify them.

An approach which is able to provide a detailed analysis of handling characteristics already in the design phase of a new car is a mathematical model of vehicle. Moreover, the detection of a potential handling problem using simulations generally requires less effort and time. In the past, many different variants of a handling model have been presented in the literature, with the emphasis being put mostly on improvements of the single-track model of a vehicle (1, 2, 3), detection of new factors affecting driving stability (4, 5, 6), or formation of complex multi-body systems (7). In all cases, the authors have tried to bring the mathematical model closer to reality, but as a consequence, the number of parameters necessary to execute such simulation has increased rapidly. Unfortunately, processes for determining values of these variables are often high in cost and time demand.

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Therefore, this article presents a mathematical model of vehicle which contains a relatively small number of parameters with the main criterion for assessing the applicability of this simplified approach being its correlation with road tests.

1. VEHICLE DYNAMICS

A single track model is used for investigations of vehicle's lateral dynamics. The presumption is that the two tyre slip angles on a single axle are the same, which makes both axles (the front and the rear) reducible into a single wheel. The motion equations quantifying the vehicle dynamics are derived from Figure 1, which shows the tyre forces, state variables and some of the basic vehicle parameters.

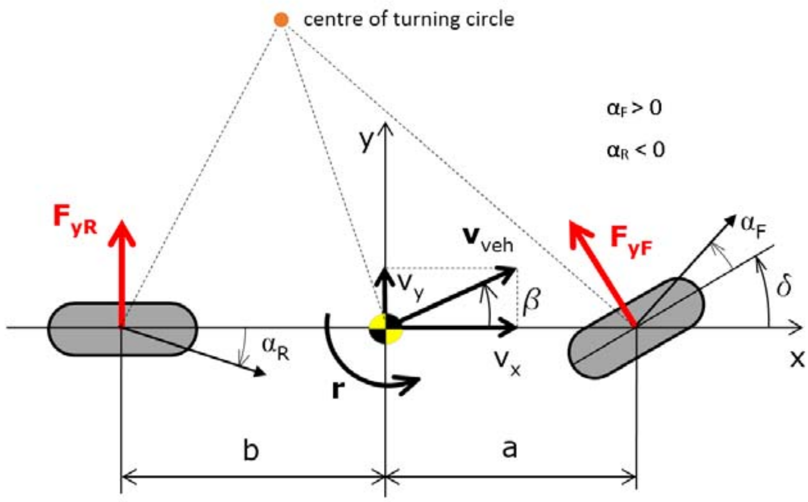

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Fig. 1 – Single track model of vehicle

The model assumes constant vehicle velocity as the sum of all the forces acting along x-axis equals zero, because traction forces compensate for the rolling resistance and the projection of the front lateral force onto the longitudinal direction. Due to this simplification, the motion of a vehicle can be described using just the equilibrium of forces in the lateral direction (y-axis) and the equilibrium of moments around the centre of gravity (C.G.). The basic differential equations of this 2DOF (two degree of freedom) model can be expressed as:

$$\sum F_y : \quad m \cdot a_y = F_{yR} + F_{yF} \cdot \cos(\delta) \tag{1}$$

$$\sum M_z : \quad I_z \cdot \dot{r} = a \cdot F_{yF} \cdot \cos(\delta) - b \cdot F_{yR} \tag{2}$$

The tyre side force F_y is generated by the elastic deformation of the tread during side slip motion, which is represented by the slip angle α. The side force-slip angle relation is close to linear for small slip angles. The side forces F_{yF} and F_{yR} can therefore be replaced by following formulae:

$$F_{yF} = C_{\alpha F} \cdot \alpha_F \tag{3}$$

$$F_{yR} = C_{\alpha R} \cdot \alpha_R \tag{4}$$

The cornering stiffness C_α, including further details of the side force-slip angle relation, is depicted in the subsequent chapter. Considering the slip angles of vehicle β to be small, the vehicle lateral acceleration and slip angles of both axles can be expressed as:

$$a_y = v_{veh} \cdot (r + \dot{\beta}) \tag{5}$$

$$\alpha_F = \arctan\left(\frac{v_{veh} \cdot \beta + a \cdot r}{v_{veh}}\right) - \delta \tag{6}$$

$$\alpha_R = \arctan\left(\frac{v_{veh} \cdot \beta - b \cdot r}{v_{veh}}\right) \tag{7}$$

Inclusion of all modifications into the motion equations leads to the resulting formulae being written as:

$$m \cdot v_{veh}\left(r + \dot{\beta}\right) = C_{\alpha R} \cdot \arctan\left(\frac{v_{veh}\beta - br}{v_{veh}}\right) + C_{\alpha F} \cdot \left(\arctan\left(\frac{v_{veh}\beta + ar}{v_{veh}}\right) - \delta\right) \cdot \cos(\delta) \tag{8}$$

$$I_z \cdot \dot{r} = a \cdot C_{\alpha F} \cdot \left(\arctan\left(\frac{v_{veh}\beta + ar}{v_{veh}}\right) - \delta\right) \cdot \cos(\delta) - b \cdot C_{\alpha R} \cdot \arctan\left(\frac{v_{veh}\beta - br}{v_{veh}}\right) \tag{9}$$

where:

a distance from front axle to centre of gravity
b distance from rear axle to centre of gravity
$C_{\alpha F}$ cornering stiffness of front axle
$C_{\alpha R}$ cornering stiffness of rear axle
I_z yaw moment of inertia
m vehicle mass
r vehicle yaw rate
v_{veh} absolute vehicle velocity
β vehicle slip angle
δ steer angle of front wheels

Numerical solution of differential equations returns a time history of vehicle motion (slip angle and yaw rate) as a response to the change in steering angle. The single track approach was formed in accordance with F.Vlk (8). All assumptions considered, this linear model can be used only for simulations of manoeuvres where the vehicle's lateral acceleration does not exceed 5 m/s^2. In this range, slip angle values of front and rear wheels stay below 3 degrees, meaning that the tyre behaviour can be considered as linear. This area of linear performance (5 m/s^2) of the vehicle may not, however, be valid for all vehicles.

2. MEASUREMENTS OF INPUT PARAMETERS

With the mathematical model completed, it is necessary to obtain the values of vehicle parameters and ensure they are as accurate as possible. Measurement of the C.G. position and the vehicle mass can be easily performed using either singular wheel or axle scale system.

Unfortunately, the processes needed to acquire some of the other inputs for the mathematical model are much more challenging, namely:

- Vehicle yaw moment of inertia
- Cornering stiffness of tyre

These vehicle properties can be reliably measured only with highly specialized equipment. The operating principle of the device for the yaw moment of inertia determination is based on the physical pendulum theory - in accordance, experimental identification of moment of inertia can be performed for example on a rotational stand which oscillates around the vertical axis, the returning motion being provided by coil springs. Such measuring stand, illustrated in Figure 2, was designed at the Institute of Automotive Engineering of Brno University of Technology. This method renders sufficiently accurate values for any passenger car in a relatively short time.

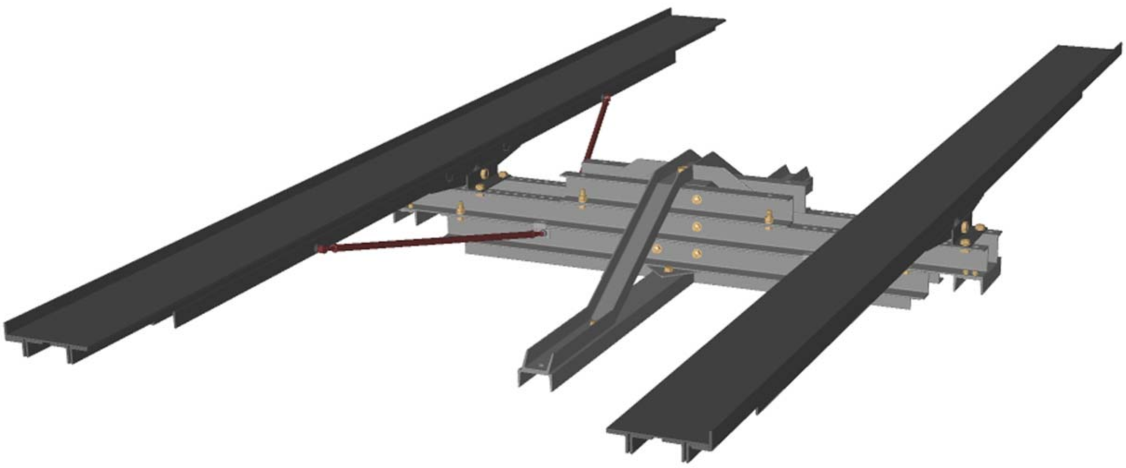

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Fig. 2 – Stand for measurements of the vehicle's yaw moment of inertia

The cornering stiffness of a tyre is the proportionality constant defined as gradient of tyre side force-slip angle relation for small angles (Figure 3). As was already mentioned, this linear relationship between lateral tyre forces and the slip angle is valid only below 3° (approximately), see Fig. 3. Determination of cornering stiffness was realized by Michelin Engineering & Services Company in France, which uses an MTS Flat-Trac specialized measuring equipment. This measuring device, as shown in Figure 4, applies pre-specified forces and motions to a tyre running on the continuous flat belt - the value of cornering stiffness can then be extracted from the data record. Because cornering stiffness is dependent on many variables, such as tyre size, wheel load, tyre inflation pressure, tread temperature, configuration of tyres used for road test validation have to correspond to the measured tyre as precisely as is practically possible.

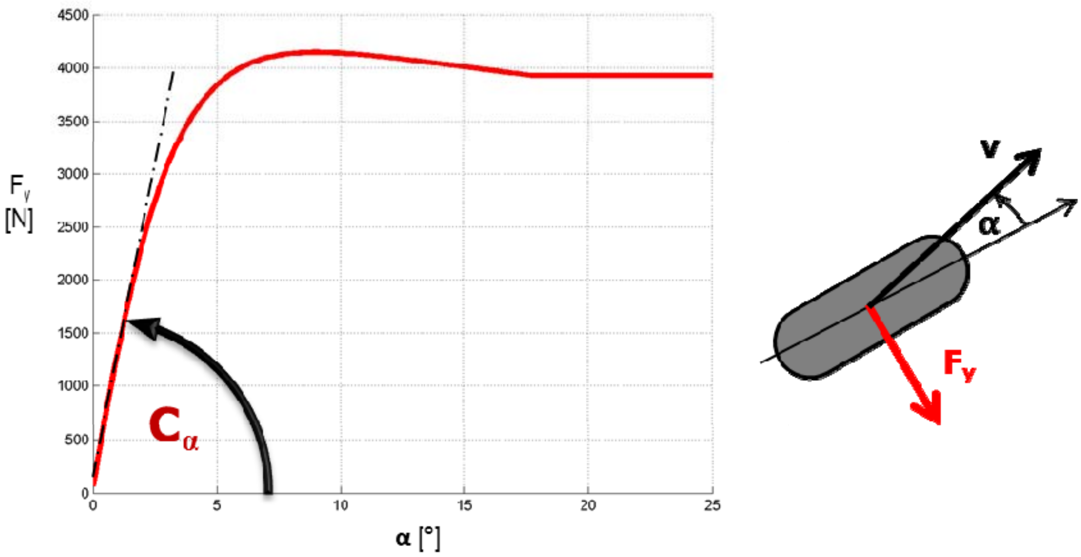

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Fig. 3 – The cornering stiffness of a tyre

Source: www.mts.com

Fig. 4 – MTS Flat-Trac III Classic tyre measurement device

3. VALIDATION OF THE MODEL

To verify the accuracy of the 2DOF mathematic model, the appropriate manoeuvre has to be chosen. In this case, the step steer input test in accordance with the ISO 7401-2003 norm was selected. This manoeuvre evaluates both steady-state and transient handling behaviour of a vehicle. Figure 5 shows the steer angle δ and the yaw rate r time traces of this test, where the steady-state phase of the manoeuvre is assessed through the steady-state value of yaw rate r_{SS} and the transient phase through the yaw rate response time t_r. These two parameters can be successfully used as criteria for evaluation of the vehicle handling (cornering). The steady-state of the manoeuvre occurs when the deviation of yaw rate stays below 5 % for at least 2s of cornering.

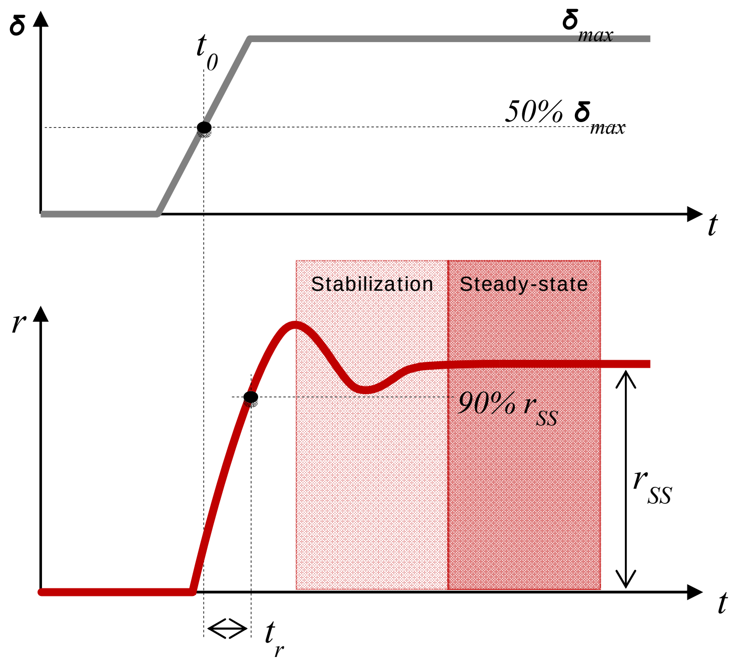

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Fig. 5 – The step steer input test: steer angle and yaw rate time traces

The next step in the validation was to conduct experimental road tests, where the following physical variables were measured: steer angle of front wheels, vehicle yaw rate, lateral acceleration and slip angle. The real manoeuvres were performed using the compact MPV-class car (multi-purpose vehicle), vehicle velocity of 80 km/h, steering wheel angle going up to 50 degrees and lateral acceleration not exceeding 5 m/s^2, with separate tests for turning to opposite directions. Another important parameter of the Step-steer input manoeuvre is the steering wheel rotation rate, with its values ranging from 300 to 350 °/s. The tests were carried out for many different configurations of the manoeuvre (changes to the steering wheel angle: 7.5°; 15°; 22.5°; …52.5°) and also with several discrete vehicle modifications (changes in mass, C.G. position, yaw moment of inertia, etc.). Figure 6 confirms that the slip angles of both axles did not exceed the limit of the linear tyre behaviour in the steady-state part of the manoeuvre. Overall, more than 100 runs in various configurations of vehicle and test were completed on a specialized polygon in Spain.

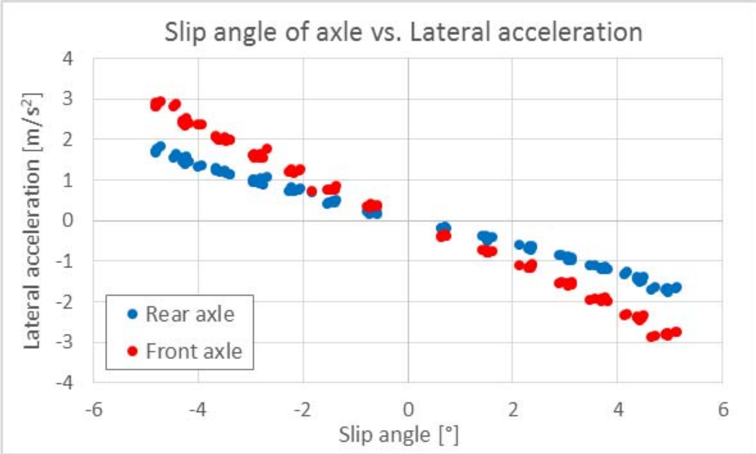

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Fig. 6 – Front and rear axle slip angle values in all performed runs

Simulations with corresponding manoeuvre conditions and vehicle parameters were performed using the aforementioned mathematical model. Thereafter, the accuracy of the model was determined by comparing the criteria values from road tests with those from simulations. The validation results are as follows:

- The relative accuracy of both the yaw rate steady state and the response time is approximately 20%.
- In general, simulations match reality very closely in the beginning phase of the step steer input test, but significant differences occur at the end of stabilising phase and in the steady-state.
- One positive conclusion is that the measured curve profile of the yaw rate is aligned with the results of simulations.

Figure 7 illustrates the comparison of steady-state yaw rate values determined from simulations and measurements. It is evident that the trend in simulation corresponds well with the measurement, proving the model works correctly in principle, just with inaccurate absolute values of the criteria. Detailed analysis pointed to irregularities in the forces on the front axle, which were probably caused by a non-infinite stiffness of the steering system.

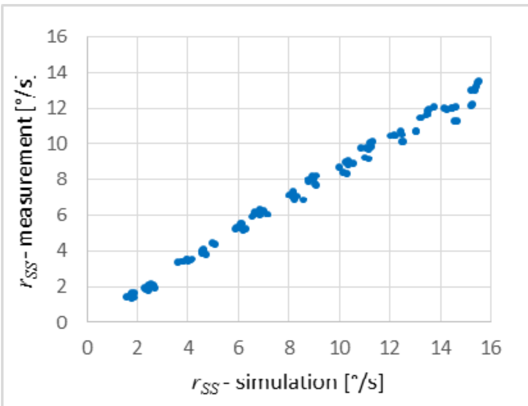

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Fig. 7 – Comparison of steady-state yaw rate values

4. MODEL IMPROVEMENT

After the accuracy was deemed unsatisfactory, the model has been enhanced with the compliance of the steering system. Integration of this factor can be realized by addition of two parameters: the wheel trail n and the steering stiffness C_S. The wheel trail is the longitudinal distance between the application point of side force and the turning centre of wheel, which is constructed as the intersection of the ground and the steering axis. Definition of the wheel trail is illustrated in Figure 6.

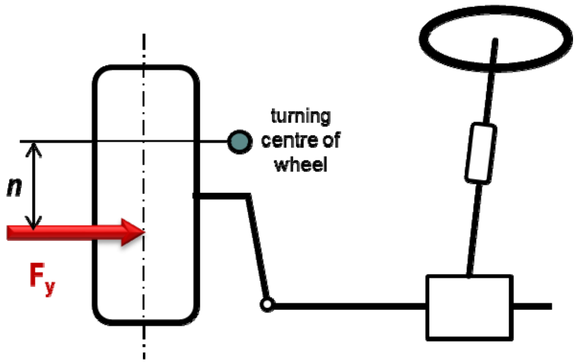

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Fig. 8 – Definition of the wheel trail of vehicle steering system

The steering stiffness describes the ability of the wheels to hold the pre-set steering angle, changes of the front wheel angle under load are caused by significant deformation in the steering system. This effect quantified by the steering stiffness value can be mathematically described through the cornering stiffness of front axle:

$$C'_{\alpha F} = \frac{C_{\alpha F}}{1 + \frac{C_{\alpha F} \cdot n}{C_S}} \tag{10}$$

However, determination of these new parameters is very complicated, and initially both values had to be estimated with the help of specialized literature (8). Simulations were then repeated with the new improved model and updated criteria values were compared with the measurements. New results of the relative accuracy (along with the initial results) are presented in Table 1, and demonstrate an increase in model accuracy for both parameters. The progress is noticeable especially for the transient phase of manoeuvre, where the deviation has decreased to 8%. The impact on the steady-states is even more pronounced: where the original model overestimates the reality by 20%, the new one underestimates it by 12%.

Tab. 1 – Relative accuracy of mathematical model versions

Criterion	Accuracy of original model	Accuracy of improved model
Yaw rate steady state value	20%	-12%
Yaw rate response time	20%	8%

Source: Author

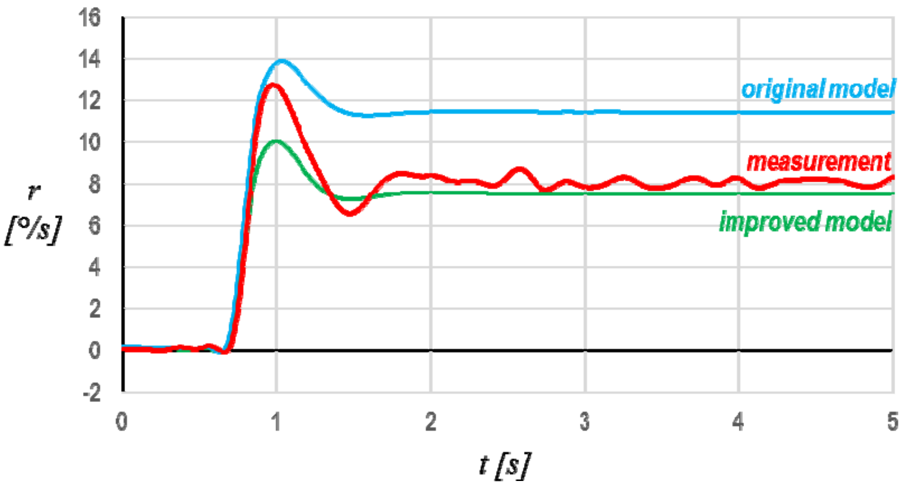

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Fig. 9 – Example of measurement-simulation correlation

Figure 10 shows the results of the original and the modified model in steady-state yaw rate values of all runs of the manoeuvre. Although the steady-state values in the improved model decreased compared to the original model, a similar linear trend in relation to the measurements is achieved.

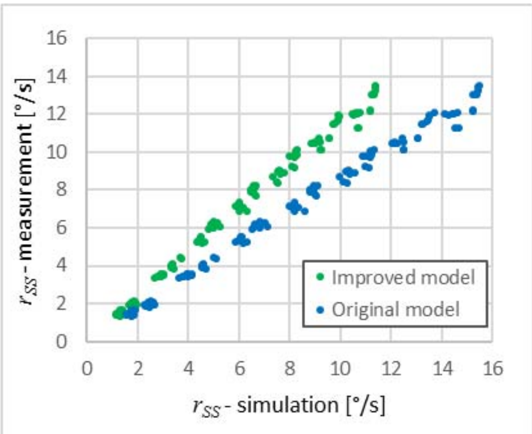

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Fig. 10 – Comparison of both model versions

The differences in results between various configurations of the vehicle were only minimal, much more significant differences were observed for changes in the manoeuvre configuration. As illustrated in Figure 11, the response time error in simulations grows with the increase of the steering angle. This might indicate that there is yet another significant factor which is currently not considered in the vehicle model (probably load transfer during cornering).

Although the improved model does not reach absolute accuracy, it is assumed that the mathematical simulation can be effectively used as a tool for vehicle handling investigations. It must not be forgotten, though, that the values of additional parameters are only estimations. The model should therefore be evaluated again after the completion of planned measurements of the steering compliance.

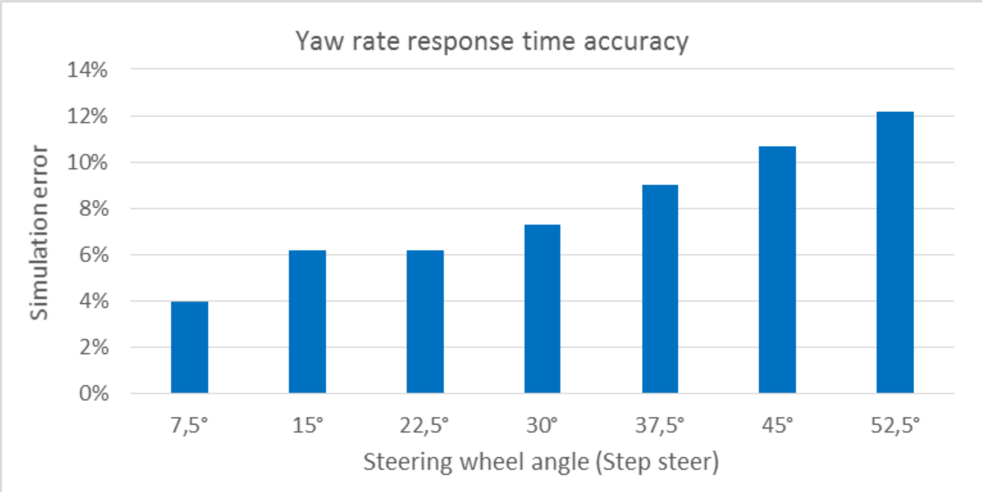

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Fig. 11 – Average response time error for different steering wheel angles

CONCLUSION

The presented mathematical model can be considered as a useful tool for vehicle handling analyses, mainly because this approach offers the possibility to assess handling already in the design phase of a new car. Initial version of the model which operated with only six parameters has achieved the relative accuracy of approximately 20% for both steady-state and transient criteria. Consequently, the model was improved by introducing the compliance of steering system parameters and its accuracy has increased to 12% and 8% for steady-state and transient response respectively. As the values of steering compliance could only be estimated at the time of writing this article, the model accuracy could still be revised. Further work will be aimed at measurements of these estimated vehicle parameters and other model improvements, especially with a focus on load transfer during cornering.

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REFERENCES

(1) KIM, J. Analysis of handling performance based on simplified lateral vehicle dynamics. *International journal of automotive technology*. 2008, Vol. 9, No. 6, s. 687-693. ISSN 1229-9138.

(2) KIENCKE, U., DAIß, A. Observation of lateral vehicle dynamics. *Control Engineering Practice*. Kidlington: Elsevier Ltd., 1997, Vol. 5, No. 8, s. 1145-1150. ISSN 0967-0661.

(3) SAVKOOR, A.R., HAPPEL H., HORKAY, F. Vehicle handling and sensitivity in transient manoeuvres. PAUWELUSSEN, J. *Vehicle performance: understanding human monitoring and assessment*. Exton, PA: Swets, c1999, s. 121-147. ISBN 9026515421.

(4) LUNDAHL, K., ASLUND, J., NIELSEN, L. Investigation vehicle model detail for close to limit maneuvers aiming at optimal control. *22nd International Symposium on Dynamics of Vehicles of Roads and Tracks*. Manchester, 2011, s. 1-6.

(5) PANG, S., GUAN, X., ZHAN, J. Research of chassis torsional stiffness on vehicle handling performance. *2010 WASE International Conference on Information Engineering: Beidaihe, China*. Los Alamitos, Calif 2010, s. 254-256. ISBN 978-0-7695-4080-1.

(6) WANG, J., HSIEH., M.F. Vehicle yaw inertia and mass independent adaptive control for stability and trajectory tracking enhancements: *2009 American Control Conference St. Louis, MO, USA*, New York: IEEE, 2009, s. 689-694. ISBN 978-1-4244- 4523-3

(7) JIANG, H., WANG, T., LI, J. Closed-Loop simulation and evaluation of vehicle handling stability on the basis simpack: *2010 Chinese control and decision conference*. New York: IEEE, 2010, s. 2748-2751. ISSN 978-1-4244-5182-1.

(8) VLK, F. *Dynamika motorových vozidel*. Vyd. 2. Brno: Prof. Ing. František Vlk, DrSc., nakladatelství a vydavatelství, 2006. 432 s. ISBN 80-239-0024-2.