

# QUESTIONS OF STAR-SHAPED LINES IN PUBLIC PASSENGER MASS TRANSPORT

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*Summary: The paper is focused on design of star-shaped lines in public passenger mass transport. The main contribution of application of these lines in integrated public passenger transport systems is seen in field of designing of supplementary lines to generic lines. Technological aspects of design and rationalizing of these lines are mentioned in the paper. Relation to transport demand models is also mentioned. Illustrational case of design of line in relation to transport demand is mentioned on the end of the paper.*

*Key words: integrated transport system, line, transport demand, public passenger mass transport.*

## INTRODUCTION

Star-shaped line could be a new challenge in the field of public passenger transport technology. Integrated public passenger transport systems are usually based on structure consisted of generic (e.g. railway) and supplementary (e.g. bus) lines.

Public passenger transport service is provided in the way that lines of supplementary subsystem are serving a quite little region in surrounding of interchanging points located on backbone generic network.

This is one of typical places for application of star-shaped lines. All places on limited area are able to be served by a number of lines, but with limited number of vehicles. Star-shaped lines are for that reason an ideal challenge for rationalization of this type of transport service, especially in the case of area with geographical obstacles to design circle lines.

## 1. DEFINITION OF STAR-SHAPED LINES

Star-shaped line is a set of lines (routes) interconnected in one point. It is able to be characterized as a set of diametric lines with intersection in one point in the case if these lines are served by one vehicle (or limited numbers of vehicles) only.

## 2. APPLICATION

There are able to be seen three main situations as appropriate for application of star-shaped structure of lines. All of these cases are also able to fade one into another.

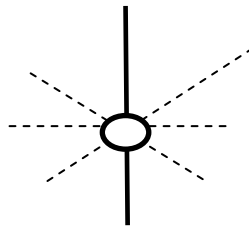
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## 2.1 Supplementary Lines to Generic Transport System

Public passenger transport system is able to be divided into generic (backbone) and supplementary (additional) subsystems interconnected in interchanging points situated on generic lines as it has been mentioned before. Structure of lines of supplementary subsystem in attraction area of certain interchanging point is able to be designed with utilizing of star-shaped lines (Fig. 1).

Application of star-shaped line in supplementary subsystem is able to be full, when all of stops in attraction area are stepwise served by one bus (by one line). Partial application is when attraction area is served by more supplementary lines (capacity or time limitations). The principle and logic of operation is the same in both cases. In second case combination of star-shaped, radial, diagonal or circle lines is possible in accordance to operational conditions as well.

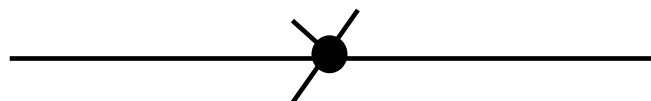


Source: Author

Fig. 1 – Supplementary Lines Connected to Interchanging Point

## 2.2 Variable Routing of Lines

Intersection point of star-shaped line is able to be occurred also in the frame of another line (e.g. radial), when the vehicle operated on line has to serve more places in close surrounding of this intersection (Fig. 2). This intersection is able to be occurred on wayside stop, but also short before end of the line. In second case this place is able to become a special case of situation mentioned in chapter 2.1.



Source: Author

Fig. 2 – Branching of Line

This solution is able to be successfully applied especially in combination with progressive technology of dial-up buses. In this case some of branches are not served in the case of any declared transport request (by phone). Operational costs are able to be reduced in this case. Problem is occurring in conception of time schedule and its travel times and dwell times. Question is dwell time extension (in the case of no transport requests on branches) or service with delay (in the case of requested service on branches), but one recommendation is clear – departure before scheduled time is inadmissible.

The main question – assessment of order for serving of branches is still remaining.

### **2.3 Off-peak Service**

The third case seen as appropriate for application of star-shaped lines is off-peak operation. Determination of off-peak time period (and of course ability of application of star-shaped lines) is depended on local transport characteristics. Night transport service or weekend transport service are able to be mentioned as most common examples of off-peak time periods when transport is organized with reduced sources (e.g. number of vehicles, staff).

### **3. ASPECTS INFLUENCING APPLICATION OF STAR-SHAPED LINES**

Factual application of star-shaped lines is able to be influenced by these aspects:

- intensities and distribution of passenger transport flows in area,
- line density of generic subsystem and frequency of its operation,
- configuration of transport network in area and geographical extent of area,
- type of utilized vehicles (capacity),
- applied fare structure,
- number of operated vehicles,
- application of progressive technologies (e.g. dial-up bus),
- declared standards of quality,
- budget for public passenger transport service.

All of these aspects are able to form final system appearance.

### **4. DESIGN AND RATIONALIZATION OF STAR-SHAPED LINES**

It is essential to define basic technological aspects for designing, planning and rationalizing of star-shaped lines.

Basic precondition is that all of branches of line have to be served within one connection on this line. Total transport output (vehicle.km) is the same in every variant of solution, because all of branches have to be served in reverse way. Question is in what order the branches may be served.

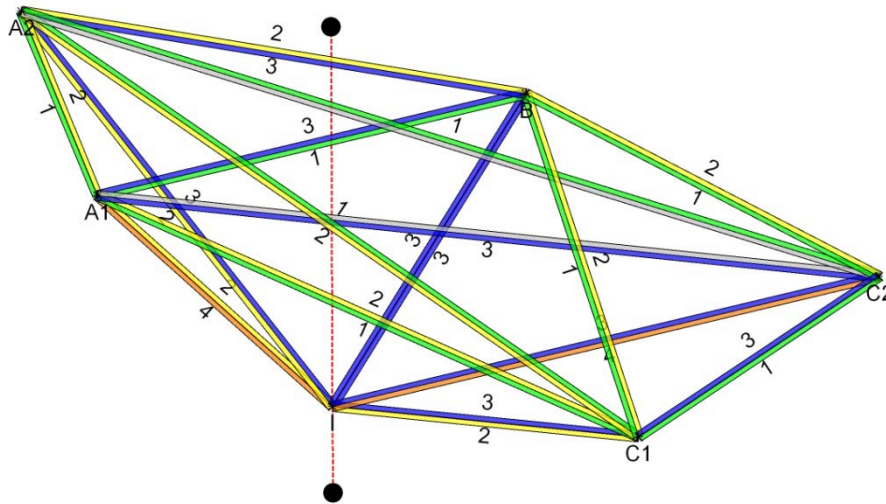
There is able to be found a relation to transport demand and to quality of transport service. Travelling in this system is able to cause specific demands (extension of trip length or travel time, in some cases fare is also able to be increased) thank to travelling through another branches of star-shaped line. This negative effect has to be minimized.

Minimizing of this effect is able in the way of selection of correct order of served branches.

There are also able to be found some dependencies on time schedule and interconnection of star-shaped supplementary lines to generic (backbone) transport subsystem (minimizing of waiting times). This is a question of time scheduling if there will be one or more connections on supplementary star-shaped line within one period of an improved frequency time schedule or to what direction of generic line connection supplementary line

will be connected. Access mentioned in this paper is able to applied in all situations in the field of time scheduling (naturally by appropriate modification according to certain situation).

OD matrix with trips numbers on every relation in the area is a basic input in this rationalization. This OD matrix is able to be got from transport survey or from validated transport model (e.g. four-step gravity transport model). Example of graphical interpretation of OD matrix for illustrational example (Chapter 5) is printed out from computer-supported transport model on the Fig. 3.



Source: Author

Fig. 3 – Graphical Interpretation of OD Matrix (sw. OmniTRANS)

Travel costs are second important input into this rationalization task. The costs are able to be expressed in monetary units, but also as a distance [km] or travel time [min]. Combination of these factors is also able to be used (transformation to monetary units).

Travel costs are calculated for every relation in solved system by the formula (1) and also for every variant of order of served branches. Costs are strictly depended on order of served branches.

$$TC_{ij} = C_{i-X} + C_{X-j} + P_w + 2 \sum_b C_b \quad (1)$$

where is:

$TC_{ij}$  travel costs for relation from stop  $i$  to  $j$  [min/km/money],

$C_{i-X}$  travel costs for connection from stop  $i$  to intersection point  $X$  [min/km/money],

$C_{X-j}$  travel costs for connection from intersection point  $X$  to destination stop  $j$  [min/km/money],

$P_w$  penalty for waiting in intersection point between services (interruption of service) [min/km/money],

$b$  index for branches served between branches with stops  $i$  and  $j$ ,

$C_b$  travel costs for whole branch [min/km/money].

The costs for travelling from/to intersection point are able to be evaluated in two different ways in dependence to selected criterions. All transport flows interchanging from/to generic subsystem have interchanging point fixed as an origin/destination of trip.

In the case of calculations with lengths (in km) only, direct way is able to be calculated only. There is a presumption that passenger will board (leave) the vehicle before (after) serving of branch where he will travel (from what he will travel). In the case of time-based costs expression (time, money), service of all branches before (or after) has to be incorporated, because extended waiting time in interchanging point will be occurred. Concrete decision of costs expression has to be made after detail analysis of concrete local conditions.

Direct costs are calculated in the case of travelling between two stops located on one branch only.

The number of variants of order is able to be represented as permutation from as many elements, how many branches are situated on line (connected to intersection point), see formula (2).

$$V = p(n_b) = n_b! \quad (2)$$

where is:

- $V$  number of order variants,
- $p(n_b)$  symbol for permutation of  $n_b$  elements,
- $n_b$  number (count) of star-shaped line branches.

For each variant of these variants it is possible to elaborate a distance matrix. Calculation of these distance matrices is able to be simplified by the way, that every branch of star-shaped line is able to create a sequence of distances (what is the same in every case, if passenger are obligated to drive through whole branch). Individual distances for every pair of origin and destination stops are able to be calculated more simply by this access according to selected order of served line branches.

Quantification of total impact on all passengers in the system is able to be evaluated in the way of scalar product between certain OD matrix and current destination matrix (matrix of costs), see formula (3).

$$\mathbf{D} = \mathbf{OD} \cdot \mathbf{L} \quad (3)$$

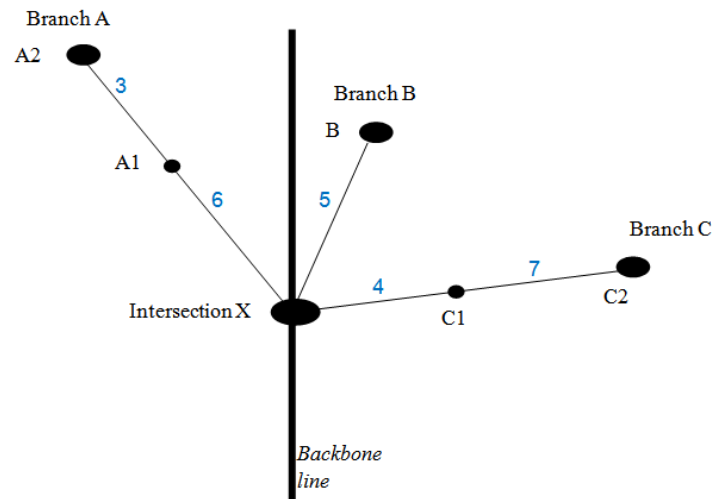
where is:

- $\mathbf{D}$  total costs of passengers [passenger.costs],
- $\mathbf{OD}$  OD matrix containing trip numbers for all origin and destination pairs [passengers],
- $\mathbf{L}$  matrix of costs for all origin and destination pairs [min/km/money].

The variant with the minimal result  $\mathbf{D}$  from this scalar product is able to be evaluated as the best, because total traffic costs of all passengers are minimized in this case.

### 5. ILLUSTRATIONAL CASE

An illustrational case has been elaborated for better explanation of the problem. There is a given transportation network with 3 branches of supplementary lines feeding the generic (backbone) line in the intersection and interchanging point  $X$  (Fig. 4). Supplementary network is able to be served by one vehicle only. Edges of the graph are characterized by its lengths in km.



Source: Author

Fig. 4 – Transportation Network for Illustrational Case

Intensities of transportation flows for certain time frame are given by OD matrix (Tab. 1 or Fig. 3). These data are able to be an output of transport survey or transport model as it has been mentioned before. Data have to be for public passenger transport mode only.

Tab. 1 – OD Matrix for Illustrational Case [Number of Passengers in Certain Time Frame]

From/To	I	A1	A2	B	C1	C2
I	0	2	3	3	2	4
A1	4	0	2	1	1	3
A2	2	1	0	3	2	0
B	3	3	2	0	1	1
C1	3	2	1	2	0	1
C2	3	0	1	2	3	0

Source: Author

Grey-marked cells are representing transport flows from/to the place where the intersection point is located, but also transport flows transferring from/to backbone line are incorporated there together.

There are 3 branches to be served by a star-shaped line in this illustrational case. It means that there are 6 possibilities how the order of served branches is able to be designed (in accordance to the formula (2)). These possibilities of branch order are  $ABC$ ,  $ACB$ ,  $BAC$ ,  $BCA$ ,  $CAB$ ,  $CBA$ .

Distance matrices incorporating travel costs (km in this case) have to be elaborated (Tab. 2 and Tab. 3). An important decision is, if penalties for service interruption in the intersection point  $X$  after serving of all sequence of branches are regarded or not. Penalties are not incorporated in mentioned case; because it is designed for optimizing of total transport costs by expression in passenger kilometres (it is equal to transport output in this case). Penalties are necessary in cases of time or money expressions of transport costs. Reduction of number of variants is possible in this case to two variants of branch serving order  $ABC$  and  $ACB$ . All of 4 other variants are equal to one of these variants by calculation without waiting penalties.

Grey-marked cells in Tab. 2 and 3 are representing values where waiting penalty is able to be occurred in the case of its incorporating for these two variants of order. In case of other orders ( $BAC$ ,  $BCA$ ,  $CAB$ ,  $CBA$ ) penalties will be occurred by other relations. For that reason all of 6 matrices have been elaborated by this way of calculation.

Tab. 2 – Distance Matrix for Order  $ABC$

From/To	I	A1	A2	B	C1	C2
I	0	6/6	9/9	5/23	4/32	11/39
A1	6/38	0	3	11	20	27
A2	9/41	3	0	14	23	30
B	5/27	33	36	0	9	16
C1	4/4	10	13	27	0	7
C2	11/11	17	20	34	7	0

Source: Author

Tab. 3 – Distance Matrix for Order  $ACB$

From/To	I	A1	A2	B	C1	C2
I	0	6/6	9/9	5/45	4/22	11/29
A1	6/38	0	3	33	10	17
A2	9/41	3	0	36	13	20
B	5/5	11	14	0	27	34
C1	4/14	20	23	9	0	7
C2	11/21	27	30	16	7	0

Source: Author

There are mentioned both possible values (for travelling to first or from last visit of interchanging point / for travelling in vehicle by all sequence of branches) in the case of trips from/to intersection (interchanging) point.

Results of scalar products calculation (regarding to the formula (3)) are mentioned in the Tab. 4. Thank to expression in kilometres result is able to be characterized as total transport costs of all passengers or as total transport output as well.

Tab. 4 – Total Transport Output [passenger.km]

Order of Branches/Expression	Way of Travel to Intersection (Interchanging) Point	
	Direct	by Sequence of Branches
<i>ABC (BCA, CAB)</i>	816	1296
<i>ACB (CBA, BAC)</i>	738	1218

Source: Author

The variant of order *ACB* is better by both expressions of criterions, because total transport output is about 6 – 10 % (depending of expression) lower than by variant *ABC*. It creates a base for optimizing of transport service in solved case.

## 6. CONCLUSION

Access to rationalization of public passenger transport service based on star-shaped lines mentioned and illustrated on very simple case in this paper is able to provide a basic idea of this optimization tasks. Proposed access is able to be characterized as an all-variant searching method from the mathematical point of view. It is also able to be programmed to the computer. Computer-supported solution will be more user-friendly than manual calculation and naturally better for acceptance in practise.

The solution is able to be extended for utilizing of more vehicles or for combination with circle lines. These extensions will be following tasks solved at the Department of Transport Technology and Control of the Jan Perner Transport Faculty (University of Pardubice).

Proposed method is also able to be incorporated in the complex optimizing procedure of transport service as a partial step. Utilizing in combination with transport models is also possible and recommended.

Practical application of star-shaped lines is also seen as appropriate in the case of design of supplementary lines connected to generic (backbone) line in integrated transport systems by serving of not so extended area with terrain obstacles and with limited number of vehicles.

## 7. ACKNOWLEDGEMENT

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