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MATHEMATICAL MODEL OF SHUNTING PROCESS AS TWO-STAGE UNRELIABLE QUEUEING SYSTEM

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Summary: The paper deals with a two-stage queuing model of the reception sidings and the hump in the marshalling yard Ostrava - Pravé nádraží. We distinguish two types of shunting - primary shunting of arriving trains we consider to be customers and secondary shunting to be system failures. By solving of the presented mathematical model some important characteristic of the modelled system are obtained.

Key words: Marshalling yard, shunting, unreliable queueing system.

INTRODUCTION

Marshalling yards are specialized railway stations. One of the main tasks of them is shunting of trains of wagons. For this purpose these stations are equipped by reception sidings in which arriving trains are being prepared for shunting; the consequential shunting process is being carried out by means of the treatment of the gravitation on a specialized facility called a hump; trains of wagons are being humped into sorting sidings.

In this paper we will model the process of trains preparation for shunting and the consequential shunting process together as a two-stage unreliable queueing system; we will model it for the marshalling yard Ostrava – Pravé nádraží, in which the reception sidings with 5 arrivals tracks, the hump and the sorting sidings are placed in series, therefore an arriving track is occupied during the whole shunting process.

In Ostrava – Pravé nádraží we can distinguish two types of shunting – primary and secondary shunting. Primary shunting represents shunting of trains arriving to the station Ostrava – Pravé nádraží, secondary shunting for example rises from the manipulation with trains of wagons incoming to the station from the industrial sidings, which come to the sorting sidings of Ostrava – Pravé nádraží. During secondary shunting a train of wagons is moved from a sorting siding into a selected reception siding via the hump and consequently it is being humped into the sorting sidings. It is obvious that primary shunting can not be performed during carrying out secondary shunting, therefore we will consider secondary shunting to be a failure of a reception siding and the hump. Therefore, we will denote trains waiting for or being in the process of primary shunting as customers and trains waiting or being in the process of secondary shunting as failures.

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Some authors developed mathematical models of these processes. In (1) there are shown some mathematical models. The authors for example presented the simple mathematical model of reception sidings as a queueing system M/M/*n*/*n* in which every arrival track constitutes a server. The authors also presented the model of both processes (the preparation for shunting and shunting) as two queueing systems placed in series; the queueing model constitutes a two-dimensional quasi birth and death process. Another two-stage queueing model was presented in paper (2). Interesting summary of railway models is introduced in paper (3). As regards above mentioned models, our model differs in assuming two types of shunting, where primary shunting represents customers and secondary shunting a system failure.

1. MATHEMATICAL MODEL

It is obvious that the both processes – the preparation of arriving trains for shunting and shunting of trains of wagons – constitute two queueing systems that influence each other. The first queueing system is formed by two homogeneous parallel placed servers with 3 places in the queue, because there are 2 crews preparing trains for shunting. The output process from the first system constitutes the input process for the second queueing system that is formed by a server shaped by the hump with the maximal queue capacity equal to 4. Both systems share the same buffer that is formed by 5 arrival tracks. Therefore, for example, if there are 5 trains in the first queueing system, then no train of wagons is prepared for shunting (or being shunted) in the second system and vice versa. And finally, the arrival track from which a train of wagons is shunted to the hump is being occupied until shunting is not finished. That is the reason, why it is necessary to model both systems as the two-stage queueing system.

Let us assume that trains arrive to the first system according to the Poisson input process with the parameter λ. Further consider that times of the shunting preparation and shunting are exponentially distributed with the parameters μ_1 and μ_2.

Secondary shunting we will consider to be failures of both systems because during secondary shunting an arrival track is occupied as well as the hump. If there are several consecutive needs of secondary shunting, then we will unite them in a single failure. Let us assume that failures occur according to the Poisson input process with the parameter η. If there already is a failure in the system, then the parameter η is equal to zero. Times to repair are exponentially distributed with the parameter ζ.

Let us establish three discrete random variables denoted as X, Y and F. The variable X describes the number of customers (trains) finding in the first system, where each customer can either wait for the service in the first system or be serviced in the first system; customers which were already serviced stay in the first system until the end of servicing in the second system. By other words the variable X expresses the number of the occupied arrival tracks. Because there are 5 arrival tracks in Ostrava – Pravé nádraží, the variable X can take the values from the set $\{0,1,2,3,4,5\}$. The variable Y expresses the number of customers finding in the second system (trains of wagons which have already been prepared for sorting on the hump or are being humped into the sorting sidings). This variable can take the values from the same set as the variable X. The last variable F can take 3 values from the set $\{0,1,2\}$, where the meaning of it is as follows:

- If the variable F is equal to 0, there is no failure (no need of secondary shunting) in the system.
- If the variable F is equal to 1, the failure of the system is waiting for repair (there is a need of secondary shunting, but the realization of it must wait due to carrying out primary shunting; the selected arrival track is indirectly occupied due to forthcoming secondary shunting).
- If the variable F is equal to 2, the failure of the system is being repaired (secondary shunting is being carried out).

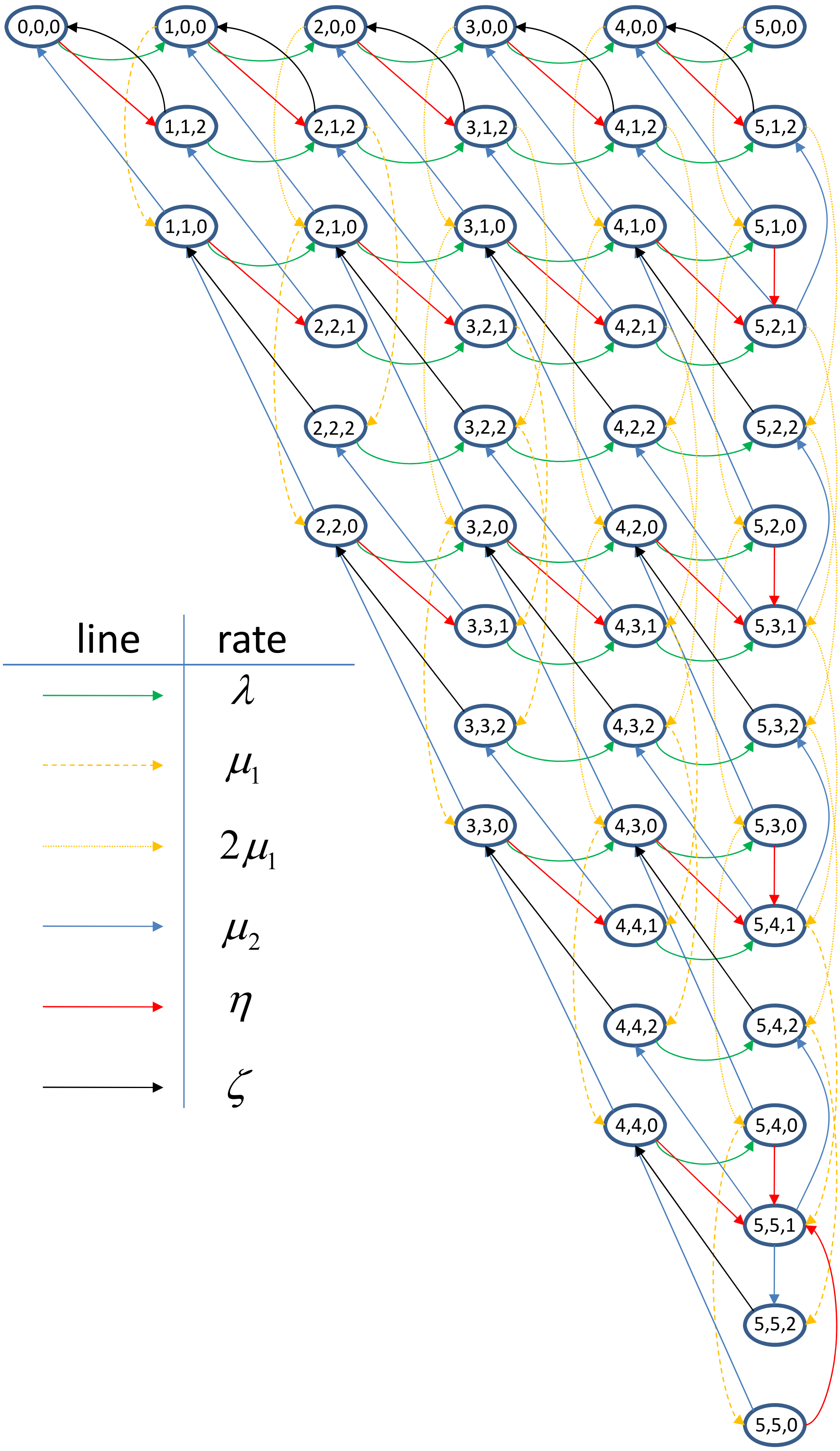
It is obvious that the individual states of the system can be described by triplets (x,y,f). The state space of the system is the union of three states subsets:

$$\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3 ,$$

where:

- The states in the subset $\Omega_1 = \{(x, y, f): x \in \{0,...,5\}, y \in \{0,...,x\}, f = 0\}$ are the states in which there are x occupied arrival tracks, y customers in the second system and there is no need of secondary shunting.
- The states in the subset $\Omega_2 = \{(x, y, f), x \in \{2,...,5\}, y \in \{2,...,x\}, f = 1\}$ are the states in which there are x occupied arrival tracks, y customers in the second system and there is a need of secondary shunting, but the realization of it must wait.
- The states in the subset $\Omega_3 = \{(x, y, f), x \in \{1,...,5\}, y \in \{1,...,x\}, f = 2\}$ are the states in which there are x occupied arrival tracks, y customers in the second system and secondary shunting is being carried out.

Let us illustrate the queueing model graphically as a state transition diagram (see in figure 1). The vertices represent the particular states of the system and oriented edges indicate the possible transitions with the corresponding rate.



Source: Author

Fig. 1 – The state transition diagram

On the basis of the state transition diagram depicted in figure 1 we can write the following linear equations that describe the behaviour of the process in steady state:

$$(\lambda + \eta)P_{0,0,0} = \mu_2 P_{1,1,0} + \zeta P_{1,1,2}, \tag{1}$$

$$(\lambda + \mu_1 + \eta)P_{1,0,0} = \lambda P_{0,0,0} + \mu_2 P_{2,1,0} + \zeta P_{2,1,2}, \tag{2}$$

$$(\lambda + 2\mu_1 + \eta)P_{k,0,0} = \lambda P_{k-1,0,0} + \mu_2 P_{k+1,1,0} + \zeta P_{k+1,1,2} \text{ for } k = 2,3,4, \tag{3}$$

$$2\mu_1 P_{5,0,0} = \lambda P_{4,0,0}, \tag{4}$$

$$(\lambda + \mu_2 + \eta)P_{k,k,0} = \mu_1 P_{k,k-1,0} + \mu_2 P_{k+1,k+1,0} + \zeta P_{k+1,k+1,2} \text{ for } k = 1,2,3,4, \tag{5}$$

$$(\lambda + \mu_1 + \mu_2 + \eta)P_{k,k-1,0} = \lambda P_{k-1,k-1,0} + 2\mu_1 P_{k,k-2,0} + \mu_2 P_{k+1,k,0} + \zeta P_{k+1,k,2}$$
$$\text{for } k = 2,3,4, \tag{6}$$

$$(\lambda + 2\mu_1 + \mu_2 + \eta)P_{k,k-2,0} = \lambda P_{k-1,k-2,0} + 2\mu_1 P_{k,k-3,0} + \mu_2 P_{k+1,k-1,0} + \zeta P_{k+1,k-1,2}$$
$$\text{for } k = 3,4, \tag{7}$$

$$(\lambda + 2\mu_1 + \mu_2 + \eta)P_{4,1,0} = \lambda P_{3,1,0} + 2\mu_1 P_{4,0,0} + \mu_2 P_{5,2,0} + \zeta P_{5,2,2}, \tag{8}$$

$$(2\mu_1 + \mu_2 + \eta)P_{5,k,0} = \lambda P_{4,k,0} + 2\mu_1 P_{5,k-1,0} \text{ for } k = 1,2,3, \tag{9}$$

$$(\lambda + \zeta)P_{1,1,2} = \mu_2 P_{2,2,1} + \eta P_{0,0,0}, \tag{10}$$

$$(\lambda + \mu_1 + \zeta)P_{2,1,2} = \lambda P_{1,1,2} + \mu_2 P_{3,2,1} + \eta P_{1,0,0}, \tag{11}$$

$$(\lambda + 2\mu_1 + \zeta)P_{k,1,2} = \lambda P_{k-1,1,2} + \mu_2 P_{k+1,2,1} + \eta P_{k-1,0,0} \text{ for } k = 3,4, \tag{12}$$

$$(2\mu_1 + \zeta)P_{5,1,2} = \lambda P_{4,0,0} + \mu_2 P_{5,2,1} + \eta P_{4,1,2}, \tag{13}$$

$$(\lambda + \mu_2)P_{2,2,1} = \eta P_{1,1,0}, \tag{14}$$

$$(\lambda + \mu_1 + \mu_2)P_{3,2,1} = \lambda P_{2,2,1} + \eta P_{2,1,0}, \tag{15}$$

$$(\lambda + 2\mu_1 + \mu_2)P_{4,2,1} = \lambda P_{3,2,1} + \eta P_{3,1,0}, \tag{16}$$

$$(2\mu_1 + 2\mu_2)P_{5,2,1} = \lambda P_{4,2,1} + \eta P_{4,1,0} + \eta P_{5,1,0}, \tag{17}$$

$$(\lambda + \zeta)P_{k,k,2} = \mu_1 P_{k,k-1,2} + \mu_2 P_{k+1,k+1,1} \text{ for } k = 2,3,4, \tag{18}$$

$$(\lambda + \mu_1 + \zeta)P_{k,k-1,2} = \lambda P_{k-1,k-1,2} + 2\mu_1 P_{k,k-2,2} + \mu_2 P_{k+1,k,1} \text{ for } k = 3,4, \tag{19}$$

$$(\lambda + 2\mu_1 + \zeta)P_{4,2,2} = \lambda P_{3,2,2} + 2\mu_1 P_{4,1,2} + \mu_2 P_{5,3,1}, \tag{20}$$

$$(2\mu_1 + \zeta)P_{5,k,2} = \lambda P_{4,k,2} + 2\mu_1 P_{5,k-1,2} + \mu_2 P_{5,k+1,1} \text{ for } k = 2,3, \tag{21}$$

$$(\lambda + \mu_2)P_{k,k,1} = \mu_1 P_{k,k-1,1} + \eta P_{k-1,k-1,0} \text{ for } k = 3,4, \tag{22}$$

$$(\lambda + \mu_1 + \mu_2)P_{4,3,1} = \lambda P_{3,3,1} + 2\mu_1 P_{4,2,1} + \eta P_{3,2,0}, \tag{23}$$

$$(2\mu_1 + 2\mu_2)P_{5,3,1} = \lambda P_{4,3,1} + 2\mu_1 P_{5,2,1} + \eta P_{4,2,0} + \eta P_{5,2,0}, \tag{24}$$

$$(\mu_1 + \mu_2 + \eta)P_{5,4,0} = \lambda P_{4,4,0} + 2\mu_1 P_{5,3,0}, \tag{25}$$

$$(\mu_1 + 2\mu_2)P_{5,4,1} = \lambda P_{4,4,1} + 2\mu_1 P_{5,3,1} + \eta P_{4,3,0} + \eta P_{5,3,0}, \tag{26}$$

$$(\mu_1 + \zeta)P_{5,4,2} = \lambda P_{4,4,2} + 2\mu_1 P_{5,3,2} + \mu_2 P_{5,5,1}, \tag{27}$$

$$(\mu_2 + \eta)P_{5,5,0} = \mu_1 P_{5,4,0}, \tag{28}$$

$$3\mu_2 P_{5,5,1} = \mu_1 P_{5,4,1} + \eta P_{4,4,0} + \eta P_{5,4,0} + \eta P_{5,5,0}, \tag{29}$$

$$\zeta P_{5,5,2} = \mu_1 P_{5,4,2} + \mu_2 P_{5,5,1}. \qquad (30)$$

Because equation (30) is a linear combination of equations (1) – (29), we omit it and replace by the normalization equation (31):

$$\sum_x \sum_y \sum_f P_{x,y,f} = 1. \qquad (31)$$

The finite equation system is formed by 46 linear equations and can be solved numerically by using software Matlab.

2. RESULTS OF EXECUTED EXPERIMENT

In table 1 there are the values of individual parameters applied in the executed experiment. These values are only the approximate estimations of the real parameters for Ostrava – Pravé nádraží. We are obliged to use these approximations because we have not yet finished the statistical processing of the real data for modelled marshalling yard.

Tab. 1 - The applied values of the random variables parameters

The parameter	λ	μ_1	μ_2	η	ζ
The applied value [h^{-1}]	1,0	0,5	3,5	0,4	1,5

Source: Author

By substitution of the parameters shown in table 1 in the equation system presented above and its solution by using Matlab we get the stationary probabilities that we need for computation of the performance measures. The reached outcomes are summarized in table 2, where *ES* means the mean number of the customers in the service, *EL* the mean number of the waiting customers and *EP* the mean number of the failures in the system.

Tab. 2 – The selected performance measures for both stages of the queuing system

The performance measure	ES_1	EL_1	ES_2	EL_2	EP_1	EP_2
The value [-]	1,57	0,84	0,22	0,19	0,20	0,18

Source: Author

On the basis of the reached outcomes we can say that the utilization of crews preparing trains for shunting is equal to 78%. The utilization of the hump is equal to 22%, but this value corresponds only to primary shunting; the overall utilization of the hump including secondary shunting is about 40%. And finally the mean number of the occupied arrival tracks including secondary shunting is equal to 3,02; we get this value as the sum $ES_1 + EL_1 + ES_2 + EL_2 + EP_1$.

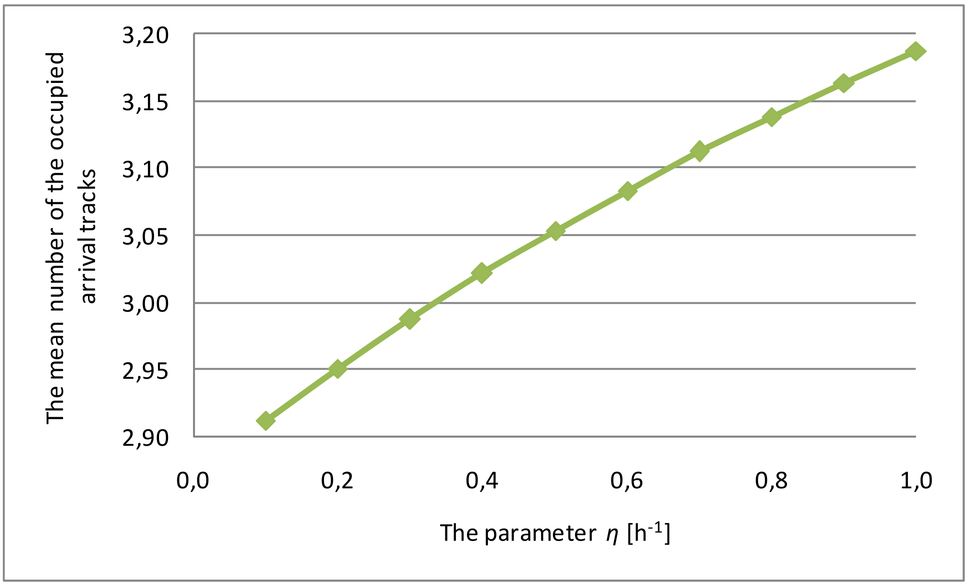
Finally, we executed some experiments in which we studied the influence of the parameter η on the performance measures. Let us consider that the parameter η can take the value from the set {0,1; 0,2; 0,3; 0,4; 0,5; 0,6; 0,7; 0,8; 0,9; 1,0} h^{-1}. The reached outcomes are summarized in table 3.

Tab. 3 – The reached outcomes

η [h^{-1}]	ES_1 [-]	EL_1 [-]	ES_2 [-]	EL_2 [-]	EP_1 [-]	EP_2 [-]
0,1	1,59540	0,93316	0,22622	0,09858	0,05871	0,05410
0,2	1,58409	0,89970	0,22305	0,13263	0,11132	0,10238
0,3	1,57414	0,87045	0,22020	0,16371	0,15866	0,14566
0,4	1,56533	0,84470	0,21761	0,19220	0,20147	0,18464
0,5	1,55749	0,82189	0,21525	0,21843	0,24033	0,21989
0,6	1,55046	0,80158	0,21309	0,24269	0,27574	0,25188
0,7	1,54414	0,78338	0,21109	0,26520	0,30812	0,28104
0,8	1,53843	0,76701	0,20924	0,28615	0,33784	0,30769
0,9	1,53324	0,75222	0,20752	0,30571	0,36519	0,33214
1,0	1,52851	0,73879	0,20591	0,32403	0,39045	0,35464

Source: Author

As regards the mean number of the occupied arrival tracks, we can see in figure 2 that this performance measure increases with the increasing value of the parameter η.



Source: Author

Fig. 2 – The impact of the parameter η on the mean number of the occupied arrival tracks

3. CONCLUSIONS

In the paper we introduced the mathematical model of the shunting process in the marshalling yard Ostrava – Pravé nádraží. We modelled the processes of the shunting preparation and shunting together as the two-stage queueing model, because both processes influence each other. As regards the following research we want to create a simulation model by using coloured Petri net in order to validate the outcomes reached by the solution of the presented mathematical model. Further we want to execute more experiments in order to get some conclusions for practice.

REFERENCES

(1) BRANDALÍK, F., KLUVÁNEK, P. Operační analýza v železniční dopravě. Bratislava: ALFA, 1984. 497 pp.

(2) HEIN, O. A Two Stage Queue Model for a Marshalling Yard. *Rail International*, 1972, Vol. 3, pp. 249-259, ISSN 0020-8442.

(3) ASSAD, A. A. Models for Rail Transportation. *Transportation Research Part A: General*, 1980, Vol. 14, No. 3, pp. 205-220, ISSN 0965-8564.