# CROSS-SECTIONAL AREA OF THE BELT CONVEYOR WITH A THREE-IDLER SET 

Leopold Hrabovský ${ }^{1}$


#### Abstract

Summary: This contribution describes a determination of maximum height of the cap of crosssectional area of the belt conveyor with one-idler set on the basis of known value of useable conveyor belt width. At the same time is determined the biggest crosssectional area of belt conveyor with three-idler set assuming that the values of surcharge angle, conveyor belt width, troughing angle and length dimensions of idlers are known as well.


Key words: cross-sectional area, belt conveyor, three roll troughed idler set

## INTRODUCTION

Maximum conveying capacity of the belt conveyor $\mathrm{I}_{\mathrm{v}}\left[\mathrm{m}^{3} / \mathrm{s}\right]$ is determined as the multiplication of the belt velocity $\mathrm{v}[\mathrm{m} / \mathrm{s}]$, slope coefficient of the belt conveyor $\mathrm{k}[-]$ and the biggest cross-sectional area $S\left[\mathrm{~m}^{2}\right]$.

The biggest cross-sectional area of the belt depends on usable conveyor belt width b [ m ] which is the function of conveyor belt width $B$ [ m ], moreover on the trough shape, i.e. on the number of idlers and their proportions (centre idler length $1_{3}$ [m] and troughing angle $\lambda$ [deg]), and on spherical cap shape of the cross-sectional area, which is limited by parabolic curve. This parabolic curve is characterized by surcharge angle $\Theta$ [deg] of the conveying material.

## 1. CROSS-SECTION AREA OF THE BELT CONVEYOR

If the value of surcharge angle $\Theta$ [deg] of conveying material is known as well as conveyor belt width B [m], it is possible to express useable conveyor belt width b [m] (see Fig. 1) according to equation (1).

$$
\begin{array}{ll}
b=0,9 . B-0,05 & \text { for } B \leq 2 m \\
b=B-0,25 & \text { for } B>2 m \tag{1}
\end{array}
$$

Distance of point $\mathbf{D}$ (see Fig. 2) from point of origin $\mathbf{0}$ of the specified coordinate system $\mathbf{x y}$ is defined as $\mathrm{b}_{1} / 2[\mathrm{~m}]$, whereas the horizontal projection of usable conveyor belt width $b_{1}[m]$ can be expressed according to equation (4). Follow from Fig. 1, for a partial dimensions $b[m]$ and $z[m]$ is valid the equation (2), for length of line segment $z_{1}[m]$ is valid the equation (3).

$$
\begin{align*}
& \mathrm{b}=\mathrm{z}+1_{3}+\mathrm{z}=2 \cdot \mathrm{z}+1_{3} \Rightarrow \mathrm{z}=\frac{\mathrm{b}-1_{3}}{2}[\mathrm{~m}]  \tag{2}\\
& \mathrm{z}_{1}=\mathrm{z} \cdot \cos \lambda=\frac{\mathrm{b}-1_{3}}{2} \cdot \cos \lambda[\mathrm{~m}] \tag{3}
\end{align*}
$$

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$$
\begin{equation*}
\mathrm{b}_{1}=2 \cdot \mathrm{z}_{1}+\mathrm{l}_{3}=2 \cdot \frac{\mathrm{~b}-1_{3}}{2} \cdot \cos \lambda+\mathrm{l}_{3}=\mathrm{b} \cdot \cos \lambda+\mathrm{l}_{3} \cdot(1-\cos \lambda)[\mathrm{m}] \tag{4}
\end{equation*}
$$

\]



Source: Author
Fig. 1 Cross-section through troughed conveyor
For the purpose of parabola construction, it is necessary to express parameter p [ m ] and parabola height $h_{\max }[m]$. From descriptive geometry is known, that if we draw tangent $\mathbf{t}$ to a parabola, see Fig. 2, and at point of intersection of parabola and tangent line (given by point D) we draw the vertical line to parabola axis $\mathbf{y}$, then is the distance between intersection of vertical line with parabola axis and normal line $\mathbf{n}$ against the tangent line $\mathbf{t}$ of parabola given by parameter size 2.p [m].

Follow from Fig. 2, $\operatorname{cotg} \Theta=\frac{2 . p}{\frac{b_{1}}{2}} \Rightarrow p=\frac{b_{1}}{4} \cdot \operatorname{cotg} \Theta[m]$
From descriptive geometry is further known, that the vertical distance of directrix $\mathbf{d}$ from parabola vertex $\mathbf{E}$ is given by size of parabola parameter p [m], as well as vertical distance of parabola focus $\mathbf{H}$ from parabola vertex $\mathbf{E}$ is given by size of parabola parameter p [ m ] because of parabola definition validity: ,,given a point (the focus) and a corresponding line (the directrix) on the plane, the locus of points in that plane that are equidistant from them is a parabola".

By using known value of parabola parameter $\mathrm{p}[\mathrm{m}]$ (equation 5) we can depict points $\mathbf{H}$, $\mathbf{E}, \mathbf{G}$, according to procedure in Fig. 3.

From Fig. 2 can be determined that the parabola intersects itself with the x -axis in point $\mathbf{D}$, whereas the distance of parabola from point of origin of the specified coordinate system is given by $b_{1} / 2[m]$. If we draw a parallel $\mathbf{r}$ to the $y$-axis, after the parabola depicting is the point of intersection of this parallel to the parabola just point $\mathbf{D}$.

Parabola will be constructed on the basis of its definition (distance of point of parabola from directrix $\mathbf{d}$ is equal to distance between focus $\mathbf{H}$ and point in question).

It stands to reason, see Fig. 2, that the joins of points HD and DI are radius vectors of point $\mathbf{D}$ of parabola (this is joint of point $\mathbf{D}$ with focus $\mathbf{H}$ and vertical line drawn at point $\mathbf{D}$ to directrix $\mathbf{d}$ ). Tangent line $\mathbf{t}$ of parabola is halving the exterior angle by means of radius vectors of its point of contact.


Source: Author
Fig. 2 - Parabola shape
Follow from Fig. 2, $\operatorname{tg} \Theta=\frac{\mathrm{h}_{\text {max }}}{\frac{\mathrm{b}_{1}}{4}} \Rightarrow \mathrm{~h}_{\max }=\frac{\mathrm{b}_{1}}{4} \cdot \operatorname{tg} \Theta[\mathrm{~m}]$
Height of parabola $\mathrm{h}_{\max }$ [m] may be expressed on the basis of Fig. 4. Let's mark the point of intersection of tangent $\mathbf{t}$ with parabola axis (y-axis) as $\mathbf{L}$, and $\mathbf{0}$ as the foot perpendicular from point $\mathbf{D}$ to the axis. Point $\mathbf{F}$ is point of intersection of surface normal with the axis $\mathbf{y}$, whereas the surface normal was constructed at point $\mathbf{D}$. Line segment $\mathbf{0 L}$ is being called the subtangent $2 . \mathrm{s}$ and line segment $\mathbf{0 F}$ the subnormal 2.p.

From descriptive geometry is known, that the vertex makes the subtangent half. Moreover, in quadrilateral LHDI are the diagonals LD and HI perpendicular to one another, the diagonal LD is halving the diagonal HI and edges DI, LH are parallel; therefore is LHDI either rhombus or square. It follows that $\mathbf{L P}=\mathbf{P D}$ and because of $\mathbf{P M} / / \mathbf{D 0}$ is $\mathbf{L M}=\mathbf{M} \mathbf{0}$. Follow from Fig. 4, EP = PM.


Source: Author
Fig. 3 - Parabola construction

General equation for a parabola is given by (7).

$$
\begin{equation*}
\mathrm{y}=-\mathrm{p} \cdot \mathrm{x}^{2}+\mathrm{h}_{\max }[\mathrm{m}] \tag{7}
\end{equation*}
$$

Substituting equation (5) into parameter p in equation (7) and substituting equation (6) into height of parabola $h_{\max }[\mathrm{m}]$ in equation (7) we get the final form of equation of parabola (8).

$$
\begin{align*}
y & =-p \cdot x^{2}+h_{\max }=-\frac{b_{1}}{4} \cdot \operatorname{cotg} \Theta \cdot x^{2}+\frac{b_{1}}{4} \cdot \operatorname{tg} \Theta=\frac{b_{1}}{4} \cdot\left(\operatorname{tg} \Theta-\operatorname{cotg} \Theta \cdot x^{2}\right)= \\
& =\frac{b_{1}}{4} \cdot\left(\operatorname{tg} \Theta-\frac{1}{\operatorname{tg} \Theta} \cdot x^{2}\right)=\frac{b_{1}}{4} \cdot\left(\frac{\operatorname{tg}^{2} \Theta-x^{2}}{\operatorname{tg} \Theta}\right)[\mathrm{m}] \tag{8}
\end{align*}
$$



Source: Author
Fig. 4 - Area formed by bulk material distribution on the belt surface

## 2. DERIVATION OF BELT CAPACITY

This derivation is performed on the basis of [2, p. 168-170]. The belt capacity is derived by simple geometry from a diagram such as Fig. 1, in which there are three rollers of equal length and where $\Theta$ [deg] is the angle of surcharge, $\lambda$ [deg] is the troughing angle, $B$ [ m ] is the width and $\mathrm{l}_{3}[\mathrm{~m}]$ is the length of the centre roller.

All calculation methods assume that the belt is filled uniformly along its length and that the load extends to within a distance $\mathrm{x}[\mathrm{m}]$ of the edge of the belt. This distance has been derived empirically and is expressed as a fraction of the belt width plus a constant, e.g.

$$
\begin{equation*}
\text { 2. } \mathrm{x}=\mathrm{B}-\mathrm{b}=\mathrm{B}-(0,9 \cdot \mathrm{~B}-0,05) \Rightarrow \mathrm{x}=\frac{0,1 \cdot \mathrm{~B}+0,05}{2}=0,05 . \mathrm{B}+0,025[\mathrm{~m}] \tag{9}
\end{equation*}
$$

The cross-section of the load consists of a trapezium ABCD and a segment of a circle AED. To calculate the area of the trapezium we need to know the length $z[m]$ along the wing idler to the limit of the load.

$$
\begin{equation*}
\mathrm{z}=\frac{\mathrm{B}}{2}-\frac{1_{3}}{2}-\mathrm{x}[\mathrm{~m}] \tag{10}
\end{equation*}
$$



Fig. 5, a


Fig. 5, b
Fig. $5-\mathrm{a}$ ) The area of trapezium, b) The area of the segment we need to know the radius R [m] of the arc AED

The area of trapezium $\mathrm{S}_{2}\left[\mathrm{~m}^{2}\right]$ is, see (11) and Fig. 5, a.

$$
\begin{equation*}
\mathrm{S}_{2}=\mathrm{S}_{21}+2 \cdot \mathrm{~S}_{22}=1_{3} \cdot \mathrm{z} \cdot \sin \lambda+2 \cdot \frac{1}{2} \cdot \mathrm{z} \cdot \sin \lambda \cdot \mathrm{z} \cdot \cos \lambda[\mathrm{~m}] \tag{11}
\end{equation*}
$$

Which reduces to, see (12).
$\mathrm{S}_{2}=\mathrm{z} \cdot \sin \lambda .\left(\mathrm{l}_{3}+\mathrm{z} \cdot \cos \lambda\right)[\mathrm{m}]$
To obtain the area of the segment we need to know the radius R [ m ] of the arc AED, which is given by (13) and Fig. 5, b.

$$
\begin{equation*}
\mathrm{R}=\frac{\mathrm{z} \cdot \cos \lambda+\frac{1_{3}}{2}}{\sin \Theta}=\frac{2 \cdot \mathrm{z} \cdot \cos \lambda+1_{3}}{2 \cdot \sin \Theta}[\mathrm{~m}] \tag{13}
\end{equation*}
$$

The area FAEDF is area of the sector, see (14).

$$
\begin{equation*}
\pi \cdot \mathrm{R}^{2} \cdot \frac{2 \cdot \Theta}{2 \cdot \pi}=\mathrm{R}^{2} \cdot \Theta\left[\mathrm{~m}^{2}\right] \tag{14}
\end{equation*}
$$

When $\Theta$ is in radians, or $\frac{\pi . \Theta}{180}$. $R^{2}$ when $\Theta$ is expressed in degrees.
The areas of each of the two triangles FA0 and FD0 are, see (15) and Fig. 1.

$$
\begin{equation*}
\frac{1}{2} . \text { R. } \sin \Theta . \text { R. } \cos \Theta=R^{2} \cdot \sin \Theta \cdot \cos \Theta\left[m^{2}\right] \tag{15}
\end{equation*}
$$

Hence the area $S_{1}\left[\mathrm{~m}^{2}\right]$ of the segment AED is (16), with application [3, page 45].

$$
\begin{equation*}
S_{1}=R^{2} \cdot \frac{\pi \cdot \Theta}{180}-R^{2} \cdot \sin \Theta \cdot \cos \Theta=\frac{1}{2} \cdot R^{2} \cdot\left(\frac{2 \cdot \pi \cdot \Theta}{180}-\sin 2 \cdot \Theta\right)\left[m^{2}\right] \tag{16}
\end{equation*}
$$

Substituting for $\mathrm{R}[\mathrm{m}]$, see aspect (13), obtaining (17).

$$
\begin{equation*}
S_{1}=\frac{1}{2} \cdot\left(\frac{2 \cdot \mathrm{z} \cdot \cos \lambda+1_{3}}{2 \cdot \sin \Theta}\right)^{2} \cdot\left(\frac{2 \cdot \pi \cdot \Theta}{180}-\sin 2 \cdot \Theta\right)\left[\mathrm{m}^{2}\right] \tag{17}
\end{equation*}
$$

And the total area $S\left[\mathrm{~m}^{2}\right]$ of the load is, see (18).

$$
\begin{align*}
\mathrm{S} & =\mathrm{S}_{1}+\mathrm{S}_{2}=\frac{1}{2} \cdot\left(\frac{2 \cdot \mathrm{z} \cdot \cos \lambda+1_{3}}{2 \cdot \sin \Theta}\right)^{2} \cdot\left(\frac{2 \cdot \pi \cdot \Theta}{180}-\sin 2 \cdot \Theta\right)+  \tag{18}\\
& +\mathrm{z} \cdot \sin \lambda \cdot\left(1_{3}+\mathrm{z} \cdot \cos \lambda\right)\left[\mathrm{m}^{2}\right]
\end{align*}
$$

## 3. CONCLUSION

In this contribution is expressed maximum height of the cap of the cross-sectional area of the belt conveyor on the basis of known value of useable conveyor belt width. At the same time is determined the biggest cross-sectional area of the belt conveyor with three roll idler set assuming that the values of surcharge angle, conveyor belt width, troughing angle and length dimensions of idlers are known as well.

## REFERENCES

(1) HRABOVSKÝ, L. Pásové dopravnỉky převyšující mezní úhel sklonu dopravy. Vysoká škola báňská - Technická univerzita Ostrava, 2005. 127 s . ISBN 80-248-0779-3.
(2) YARDLEY, E. D, STACE, E. D. Belt conveying of materials. Woodhead Publishing Limited and Maney Publishing Limited on behalf of The Institute of Materials, Minerals \& Mining. 2008, 189 s. ISBN 978-1-84569-230-8.
(3) DLOUHÝ, Z., HRUŠA, K., KŮST, J., ROHLÍČEK, J., TAIŠL, J., ZIERIS, J. Úvod do matematické analýzy. Státní pedagogické nakladatelství Praha, 1983. 472 s . ISBN 14-25283.


[^0]:    ${ }^{1}$ doc. Ing. Leopold Hrabovský, Ph.D., VŠB - Technical University of Ostrava, Faculty of Mechanical Engineering, Institute of Transport, Department of Transport and Process Equipment, 17. listopadu 15/2172, 708 33, Ostrava - Poruba, Tel.: +420 597323 185, Fax: +420 596916 490, E-mail: leopold.hrabovsky@vsb.cz

