DETERMINING THE SPEED OF VEHICLES BEFORE AND AFTER CRASH

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Summary: The article is representing typical example of crash analytical solution. There are presenting basic principles and their calculations in the area of vehicle motion during the accident.

Key words: Crash, Analytical solution, Traffic safety.

INTRODUCTION

In road traffic accidents occur on a different type. It is a vehicle crashes - a barrier, the vehicle - pedestrian, vehicle - vehicle, or a combination thereof. After impact, the movement of vehicles varies. In order to determine the accident, it is important to know the facts on which it is determined by the initial speed of the vehicle.

The impact is action where the interfacing of two or more bodies in the short term significantly change velocity at least one of the bodies. We talk about the process of mechanical action of large forces at short notice. In this process the speed of point objects are changed. It is obvious that the size of the common contact area is changing too during the life of character.

1. CENTRIC IMPACT

Impact force direction is line perpendicular to the touchpad, which passes through center of gravity of this area. Has the same direction as the impact force.

Impact force is divided according to the position of impact force:

- **CENTRIC IMPACT** - occurs when the impact force direction passes through the centers of gravity of interfacing bodies
- **ECCENTRIC IMPACT** - occurs when the impact force direction do not traverse through the centers of gravity of interfacing bodies.

CENTRIC IMPACT could be:

straight line impact - velocity vectors lie on the same line as a impact force direction,

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bevelled impact - velocity vectors do not lie on the same line as a impact force direction.

During the two phases of impact force increasing to the maximum until an equal velocity of both vehicles will be reached, then decreases to zero. As defined by Newton the impact of bodies can be divided into both phases: „compression“ and „restitution“.

1.1 Compression phase

Consequence of impact force increasing is body deformation (ram parts of vehicles at the impulse point are identical. The centers of gravity of interacted vehicles are therefore much closer. Compression phase ends at the moment the car body deformation reaches a maximum "value. At this stage, the kinetic energy is converted into a car deformation work.

1.2 Phase of restitution

- being from the moment of maximum deflection and the time separation. In this part of the deformation energy converted into kinetic energy of the car, the distance between the axes of cars is increasing and bodies (vehicles) are trying to restore its original shape. Because the materials have certain physical characteristics, complete shape recovery does not occurs. After the crash remains a permanent deformation. It is important to define the change in impact strength over time, which is the **impulse impact strength**. Impact force decreases and the velocity of the vehicles are different.

$$I_R = \int_0^t \vec{F}_R dt = I_{RK} + I_{RR} \tag{1}$$

I_{RK} - Impulse impact strength in compression phase [N]

I_{RR} - Impulse impact strength in phase of restitution [N]

t_K - compression phase time [s]

t_R - restitution phase time [s]

t - Impact time, $t = t_k + t_R$ [s]

I_R - Impulse impact strength [N]

F_R - Impact strength [N]

1.2.1 Coefficient of Restitution

The ratio of impact strength I_{RR} pulse applied to the body in restitution to the stage of Impulse impact strength I_{RK} forces acting on the body at the stage of compression is called the coefficient of restitution known **k**.

Due to elasticity of the vehicle structure, the two vehicles will separate again. The coefficient of restitution is defined as ratio between restitution I_{RR} and compression I_{RK} impulse.

$$k = \frac{I_{RR}}{I_{RK}} \tag{2}$$

The identification of coefficients of restitution in vehicle to vehicle collisions is impractical since each vehicle to vehicle combination has its unique restitution response. Vehicle to barrier coefficients of restitution can be measured for specific vehicles. Coefficients of restitution between two vehicles for which the vehicle to barrier coefficients of restitution are known may be predicted. Expressed in a more useful and more common form, the coefficient of restitution ***k*** is the ratio of the post (after)-impact separating velocity v_{2a} - v_{1a} of the colliding bodies to their (before) pre-impact closing velocity v_{1b} - v_{2b} .

$$k = \frac{v_{2a} - v_{1a}}{v_{1b} - v_{2b}} \tag{3}$$

The coefficient of restitution varies from zero for a perfectly plastic impact to unity for a perfectly elastic collision, and has been shown to depend upon the impact velocity and the shape and size of the colliding bodies. The coefficient of restitution lies in the range between 0.1 and 0.3 in real vehicle to vehicle collisions. Bumper-to-bumper collisions at low closing velocity are primarily elastic. The bumpers deform to some degree during impact and then rebound to nearly their pre-impact condition, coefficient of restitution values higher than 0,3.

Theoretically, there may be the following cases:

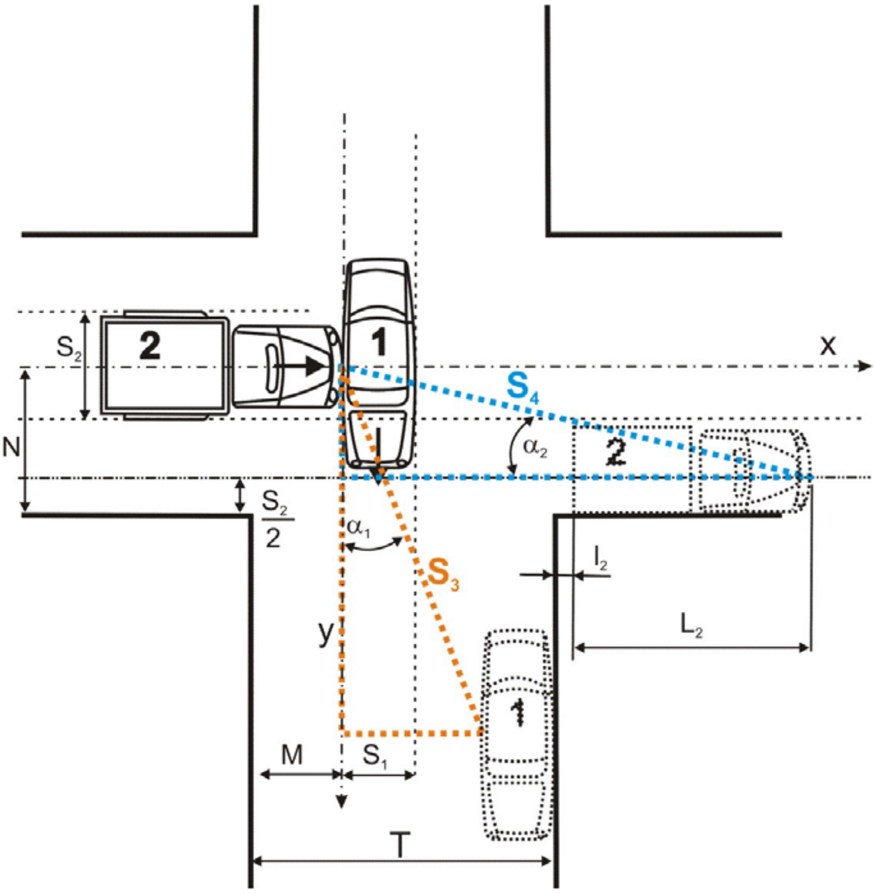
1. k=1, elements are completely flexible - there is perfectly elastic character
2. k=0, perfectly plastic character
3. Practically, the coefficient of restitution in the range 0 <k <1, it is an imperfect elastic character - the most common case

Based on the theory of Impact is possible to determine the speed of motor vehicles before and after the collision of the:

- law on the conservation of linear momentum,
- conservation of energy,
- other laws of physics.

2. DETERMINATION OF SPEED ON CRASH BETWEEN TWO CARS

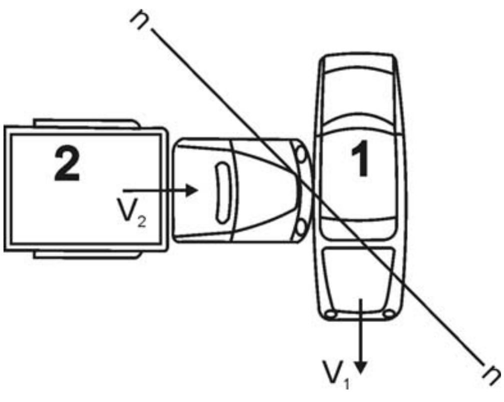
Procedure for solving the solution can be documented incidents of fig.1. At the crossroads with no vertical traffic signs crashed lorry (2) in a passenger car (1). A footprint of the accident site secured by transport police after a car crash changed the direction of travel, went six meters at an angle and hit the left wheel to curb road.



Source: Authors

Fig 1 - Accident floorplan

Lorry (2) after the crash also changed its course, and stood up in its final position. There are recorded tracks during braking of the truck after the collision. Usually this distance is measured. According the police file any serious damage has been occurred on the truck (2). There is appeared door deformation on vehicle (1). It is possible to conclude impact of vehicles had elastic character. Vehicle (2) was carrying 2,782kg of cargo. The status of the surface was paved and dry. To determine the initial velocity of vehicles is necessary to find out angles α1 and α2.



Source: Authors

Fig 2 - Scheme of collision

According the scheme fig.2 eccentric bevelled impact arises and coefficient of restitution is negative. The accident floor-plan is calculated the angle of speed vector of the car to the impact strength vector n:

$$Sin\alpha_1 = \frac{T-(M+S_1)}{S_3} = \frac{7-(2+1{,}52)}{6} = 0{,}58\,; \quad => \quad \alpha_1 = 35^0 28' \tag{4}$$

T - width of the roadway in the direction of movement of vehicle (1); T = 7m;

M = 2m;

S_1 - width of vehicle (1); S_1 = 1,52m;

S_3 - bevelled track of vehicle (1) gravity movement ; S_3 = 6m.

Now calculate the angle of velocity vector of the lorry (2) and impact force direction vector:

$$tg\alpha_2 = \frac{N-\frac{S_2}{2}}{T-M+l_2+L_2} = \frac{4-\frac{2{,}2}{2}}{7-2+0{,}5+5{,}7} = 0{,}261\,; \; => \quad \alpha_2 = 14^0 38' \tag{5}$$

N - distance from the right roadside and longitudinal axis of the lorry (2); N = 4m;

S_2 - width of the lorry (2); S_2 = 2,2m;

L_2 - length of the lorry (2); L_2 = 5,7m; roadway

l_2 - distance to the rear of the vehicle from roadway; l_2 = 0,5m;

The angles α1 and α2 is necessary to establish precisely because of their basis is determined by the movement of vehicles after the collision. Track motion of the lorry (2) gravity from the collision to final position can be determined by the following equation:

$$S_4 = \sqrt{(7-2+0{,}5+5{,}7)^2 + \left(4-\frac{2{,}2}{2}\right)^2} = 11{,}55\,\text{m}; \tag{6}$$

Next, we need to address both vehicles in one reference frame. Through the impact point create y - axis defined in the direction of movement of a vehicle (1) and x - axis defined in the direction of lorry (2) motion.

According to the theory of impulse of impact strength for vehicle (1):

$$m_1\upsilon_1' - m_1\upsilon_1 = -\int_0^t F_R dt\,; \tag{7}$$

for lorry (2):

$$m_2\upsilon_2' - m_2\upsilon_2 = \int_0^t F_R dt\,; \tag{8}$$

When a collision is fulfilled law of conservation of linear momentum:

$$m_1\upsilon_1' + m_2\upsilon_2' = m_1\upsilon_1 + m_2\upsilon_2\,; \tag{9}$$

Based on the previous equations we use decomposition into x and y coordinates, we get:

$$m_1\upsilon_{1.x}' + m_2\upsilon_{2.x}' = m_1\upsilon_{1.x} + m_2\upsilon_{2.x}\,;$$

$$m_1\upsilon_{1.y}' + m_2\upsilon_{2.y}' = m_1\upsilon_{1.y} + m_2\upsilon_{2.y}\,.$$

υ - velocity of vehicles before collision [m.s^{-1}]

υ' - velocity of vehicles after collision [m.s^{-1}]

m_1 - mass of vehicle (1); $m_1 = \dfrac{G_1}{g} = \dfrac{16700}{9{,}81} = 1700kg$;

G_1 =16700 N – gravity of vehicle (1) including passengers, drivers and storage;

$g = 9.81 m.s^{-2}$ – gravitate acceleration;

$G_2 = 56320$ N – immediate weight of lorry (2), including passengers, drivers and storage, the mass is 2782kg;

m_2 - lorry mass (2), $m_2 = \dfrac{G_2}{g} = \dfrac{56320}{9.81} = 5750kg$;

$\upsilon_{1.x}$ - component speed of vehicle (1) before the collision axis x ;

$\upsilon_{1.y}$ - component speed of vehicle (1) before the collision axis y ;

$\upsilon_{2.x}$ - component speed of lorry (2) before the collision axis x ;

$\upsilon_{2.y}$ - component speed of lorry (2) before the collision axis y ;

Velocity of vehicle after collision may be determined using energy conservation law. Comparing the kinetic energy and work consumed by sideways skid of the vehicle (1) and also work consumed in changing the amount of center of gravity after impact.

Center of gravity is moved over a distance S3 from the original direction inclined at an angle α1:

$$\frac{m_1 \upsilon_1'^2}{2} = \frac{G_1 \upsilon_1'^2}{2g} = G_1.S_3.\mu_b + G_1.h \tag{10}$$

Modified the equations (10) we obtain the relationship to determine the speed of a car after a collision in the form:

$$\upsilon_1' = \sqrt{2 \cdot g \cdot \left(S_3.\mu_b + h\right)} \tag{11}$$

$S_3 = 6m$ - distance of beveled motion of center of gravity from the collision point to the final position of the vehicle (1);

$\mu_b = 0{,}6$ - coefficient of wheel adhesion (road surface was dry asphalt);

h - change in the center of gravity due to tilt a car collision [m]; $h = h_g^{'} - h_g$

h_g – amount of vehicle´s center of gravity [m],

$h_g^{'}$ - maximum amount of center of gravity on the border overturning [m]

To what level is possible rise center of gravity of vehicle (1) (without threatening its flip)? We can determine from equation of moments around the axis of the wheel with tendency to flip a car, FIG. 3. We can write equation

$$m \cdot b \cdot h_g^{'} = m \cdot g \cdot x \tag{12}$$

x – is the horizontal distance from the center of gravity tilting edge [m]

b – vehicle motion deceleration [m.s^{-2}]

It can be determined as a function of lateral grip of tire and angle α_1, which takes into account diversion of vehicle movement away from the original direction of movement

$$b = g \cdot \mu \cdot \sin \alpha_1 \tag{13}$$

substitute b for a deceleration in equation (3) and the modification we are getting:

$$\frac{h_g^{'}}{x} = \frac{g}{g \cdot \mu \cdot \sin \alpha_1} \tag{14}$$

Fraction on the left side of the equation is a function of tan γ. Value of a fraction on the right side we can calculate. Valid tan γ = 2,873.

It is easy to determine that the angle γ has a value of 70.81 °. Based on the value of angle γ is impossible to determine the maximum allowable amount of center of gravity, because the size of hypotenuse c right triangle in Fig. 3 is the distance from the center of gravity tilting edge (if we neglect the deformation of the wheel) and it can be calculated using the Pythagorean theory:

$$c = \sqrt{\left(\frac{S_1}{2}\right)^2 + h_g^2} \tag{15}$$

The maximum allowable center of gravity is determined by the equation:

$$h_g^{'} = c \cdot \sin\gamma = \left(\sqrt{\left(\frac{S_1}{2}\right)^2 + h_g^2}\right) \cdot \sin\gamma \tag{16}$$

Substituting into equation (4) allowable level of gravity reaches

$$h_g^{'} = \left(\sqrt{\left(\frac{1{,}52}{2}\right)^2 + 0{,}65^2}\right) \cdot \sin 70{,}81° = 0{,}94m \tag{17}$$

using equation $h = h_g^{'} - h_g$ we find that the focus during the accident could raise their position on

$h = 0{,}94 - 0{,}65 = 0{,}29m$. Substituting the speed of a car after the collision is:

$$\upsilon_1' = \sqrt{2 \cdot g \cdot (S_3.\mu_b + h)} = \sqrt{2 \cdot 9{,}81 \cdot (6.0{,}6 + 0{,}29)} = 8{,}74 \text{ m.s}^{-1} = 31{,}46\text{km.h}^{-1}.$$

Now we determine the speed of the lorry (2) after the collision using the law of conservation of kinetic energy:

$$\frac{G_2.\upsilon_2'^2}{2g} = G_2.f_1.S_4 \tag{18}$$

$S_4 = 11{,}55$ m – vehicle´ s (2) centre of gravity distance from the collision point to its final position;

$f = 0{,}02$ - coefficient of friction;

final velocity of vehicle determination after the collision of vehicle (2) after adjusting:

$$\upsilon_2' = \sqrt{2 \cdot g \cdot f_1 \cdot S_4} = \sqrt{2 \cdot 9{,}81 \cdot 0{,}02 \cdot 11{,}55} = 2{,}13 \text{ m.s}^{-1} = 7{,}66\text{km.h}^{-1}$$

Now it is necessary to determine initial velocities before the collision. We assume as a lorry before the crash did not pass in the direction of axis y.

Therefore component velocity of vehicle (1) for y - axis after the collision is:

$$\upsilon_{1.y}' = \upsilon_1' \cos\alpha_1 ; \quad \upsilon_{2.y}' = \upsilon_2' \sin\alpha_2 ; \tag{19}$$

After treatment in the previous equation:

$$m_1\upsilon_{1.y} = m_1\upsilon_1'.\cos\alpha_1 + m_2\upsilon_2'.\cos\alpha_2 \tag{20}$$

whence we get such a condition and find the initial velocity of vehicle (1) before the collision:

$$\upsilon_{1.y} = \frac{m_1\upsilon_1' \cos\alpha_1 + m_2\upsilon_2' \sin\alpha_2}{m_1} = \frac{1700.8{,}74.0{,}8138 + 5750.2{,}13.0{,}25}{1700} = 8{,}91 \text{m.s}^{-1} = 32{,}1$$

km.h^{-1}.

We can determine the initial velocity of the lorry (2) before the collision in the same way.

$\upsilon_{1.x} = 0$, we assume as a vehicle (1) before the crash did not pass in the direction of the x-axis.

Therefore the calculation of x-wall component of the final velocity after the collision is:

$$\upsilon'_{1.x} = \upsilon'_1 \sin\alpha_1; \quad \upsilon'_{2.x} = \upsilon'_2 \cos\alpha_2; \tag{21}$$

After treatment in the previous equation:

$$m_2\upsilon_{2.x} = m_1\upsilon'_1.\sin\alpha_1 + m_2\upsilon'_2.\cos\alpha_2; \tag{22}$$

whence we find the initial velocity of the lorry (2) before the collision:

$$\upsilon_{.2.x} = \frac{m_1\upsilon'_1 \sin\alpha_1 + m_2\upsilon'_2 \cos\alpha_2}{m_2} = \frac{1700 \cdot 8{,}74 \cdot 0{,}58 + 5750 \cdot 2{,}13 \cdot 0{,}9676}{5750} = 3{,}58\,\mathrm{m.s^{-1}}$$

= 12,9 km.h^{-1};

In this case, according to our calculation values of initial velocity before the collision are in vehicle (1) 33km.h^{-1} and lorry (2) 12,9 km.h^{-1}. The accident record is no indication as to the braking distance of the lorry (2). The driver in the notice says that before the crash he could not push the brake pedal, which, according to calculations is not true. Based on the observed data we conclude that the lorry (2) is hampered and went free motion after the crash in the final position. It is impossible to ignore testimony of the lorry driver (2) on his brakes. The absence of braking marks, does not assume full braking effect. If the argument about braking of the lorry was true, the initial velocity of (2) would be higher than 12,9 km.h^{-1}. In the case of vehicle (1) we know center of gravity lateral shift from the collision to the final position. If you would notice the driver was true, the initial velocity of the vehicle (1) before impact had to be greater than 31,46 km.h^{-1}. The final location of lorry (2) would be further than the calculated values.

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