PARALLEL COMPUTING OF CROSS AMBIGUITY FUNCTION WITH BRUTE FORCE METHODS

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Summary: In this paper we describe brute force methods of cross ambiguity function (CAF) which is key function for successful location tracking for passive radar systems. The CAF depends upon the direct and the reflected signals from targets. The CAF represents the power spectral density function of the cross correlation between direct and reflected signals from the target. The computation of CAF in real-time is a challenging task for parallel programming techniques.

Key words: cross ambiguity function, parallel computing, passive radar system, signal processing

INTRODUCTION

Most of the modern radar systems work on the principle of transmitting radar signals to space and then receiving reflected signal from the target which contains the information about the target. From the reflected signal we can extract information about location, speed, and the type of the target. The primary disadvantage of radar systems based on this principle is that it can be easily detected by other electronic devices due to the transmitted signal. Another alternative to these “active” radar systems is passive radar systems (aka passive location systems, aka passive coherent location - PCL). The PCL systems track objects by processing reflections from non-cooperative sources of illumination in environment. The non-cooperative illuminators can be transmitters of FM, AM, DAB, analog and digital television broadcasting, cellular phone base station, etc. (1, 4, and 6).

The PCL systems are special cases of bistatic radars (3, 5) which have divided transmitting and receiving part. The principle of detection is based on “comparing” direct signal $s_1(t)$ from transmitter and reflected signal $s_2(t)$ (included information about targets) from the target. We can determine transmitter – target – receiver triangle from the known position of the transmitter, the receiver, the direct signal and reflected signal, called bistatic triangle. The speed of signal propagation is equal to the speed of light. From known distance between transmitter and the receiver, we can determine the time of propagation for this distance. The time of propagation between transmitter – target – receiver can not be calculated directly, because position of the target is unknown. Solution to this problem is based on measuring time difference between direct signal and reflected signal. This method allows that bistatic range can be determined by the use of cross ambiguity function (CAF). The CAF provides not only the position of targets but also the speed of targets. The speed of target is

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calculated from frequency shift between direct and reflected signal (Doppler shift). The Doppler shift is defined as:

$$f = f_0\left(\frac{c}{c \pm v_c}\right) \tag{1}$$

where f is the frequency of the reflected signal, f_0 is the frequency of original signal, c is the speed of light and v_c is the speed of the target.

The detection of the target depends on computing two based variables:

- Time delay $\tau\ [s]$ between direct and reflected signal. From these values we can compute distance between the receiver and the target.
- Doppler shift $f\ [Hz]$ between direct and reflected signal from this we can compute speed of the target.

The time-continuous mathematical function to determine time delay and Doppler shift is called **C**ross **A**mbiguity **F**unction [1]

$$CAF(\tau, f) = \int_0^T s_1(t)\, s_2^*(t+\tau)\, e^{-j2\pi ft} dt, \tag{2}$$

where T is total integration time, $s_1(t)$ is direct signal at analytic form (complex), $s_2(t)$ is reflected signal at analytic form (complex), * is complex conjugate.

The discrete-time form of CAF [1] is

$$CAF(\tau_d, f_d) = \sum_{n=0}^{N} s_1[n]\, s_2^*[n+\tau_d]\, e^{-j2\pi f_d \frac{n}{N}} \tag{3}$$

where N is total number of samples, n is *n-th* sample, $s_1[n]$ is discrete-time direct signal in complex analytic form, $s_2[n]$ is discrete-time reflected signal in complex analytic form, * is complex conjugate of the signal, τ_d is time delay, f_d is Doppler shift. From now in this paper we will use only discrete time form of CAF and for simplification of time delay and Doppler shift will be formally described by $\tau = \tau_d$ and $f = f_d$. Time delay and Doppler shifts are defined by group of values

$$\tau = \{\tau_p\}_{p=1}^{N_\tau}$$
$$f = \{f_q\}_{q=1}^{N_f} \tag{4}$$

where N_τ is total number of time delays and N_f is total number of Doppler shifts. N_τ is defined by maximal range of radar $R_{\max}$ and depends on range resolution dR while N_f is defined by maximum speed of target and depends upon Doppler resolution dF. Range resolution and Doppler resolution are defined by

$$dR = cT = \frac{c}{f_s} \qquad dF = f_s / N \tag{5}$$

where c is speed of light and f_s is the sampling frequency.

The primary challenge is that the computation time of $CAF(\tau, f)$ which rapidly increases with the requested resolution of CAF (typically $dF = 1\ Hz$). Hence the time of computation increases rapidly with the number of samples (this will be described below for each method of computation of CAF). Typical values of number of samples are $N = 2^{17}\ to\ 2^{19}$. Higher resolution of CAF is needed to determine the target position accurately. In this paper we describe the brute methods to calculate the cross ambiguity function, without any loss in resolution. These methods will also be compared in their time complexity in terms of their dependence over the number of cores in processor unit (CPU) and also in number of mathematical floating point operations per second. The time of computation is critical factor for using this system in real-time applications. The requested time for use at real-time application is approximately 1-2 s. CAF is a function of two variables (time delay and Doppler shift). The CAF can be transformed from $CAF(\tau, f)$ to $CAF(R, f)$. For better visualization (Fig. 1) we can transform time delay into range using the expression $R = \{R_p\}_{p=1}^{N_\tau} = \{\tau_p c\}_{p=1}^{N_\tau}$.Example of CAF is on Fig. 1.

The time of computation of CAF can be decreased by:

- Decreasing number of samples (unwanted from reason with decreasing resolution of radar),
- Parallelization of computation on CAF M-cores CPU,
- Parallelization of computation CAF on cluster (future work),
- Parallelization of computation CAF on GPU (future work).

1. BRUTE FORCE METHODS OF CAF COMPUTATION

1.1 The summation method of CAF

The summation method (2) is based on direct computation equation (3). The total amount of mathematical operations (7) in defined range N_τ and N_f is defined by

$$P_{sum} = 12 N_\tau N_f N \text{ [flops]} \tag{6}$$

The disadvantage of this method is that it needs two loops for computation. Its performance degrades very rapidly with the increase in the number of samples. The total amount of mathematical operation increases with N_τ and N_f too, but N_τ and N_f are obviously very small subset of N, $N_\tau, N_f \ll N$. In chapter 3 we will show that total amount of flops required for typical passive radar system.

However, the advantage of summation method is that we can compute CAF for any values of time delays and Doppler shifts without any limitation. This advantage is very important because we can use this method for computing any resolution in range or Doppler shift.

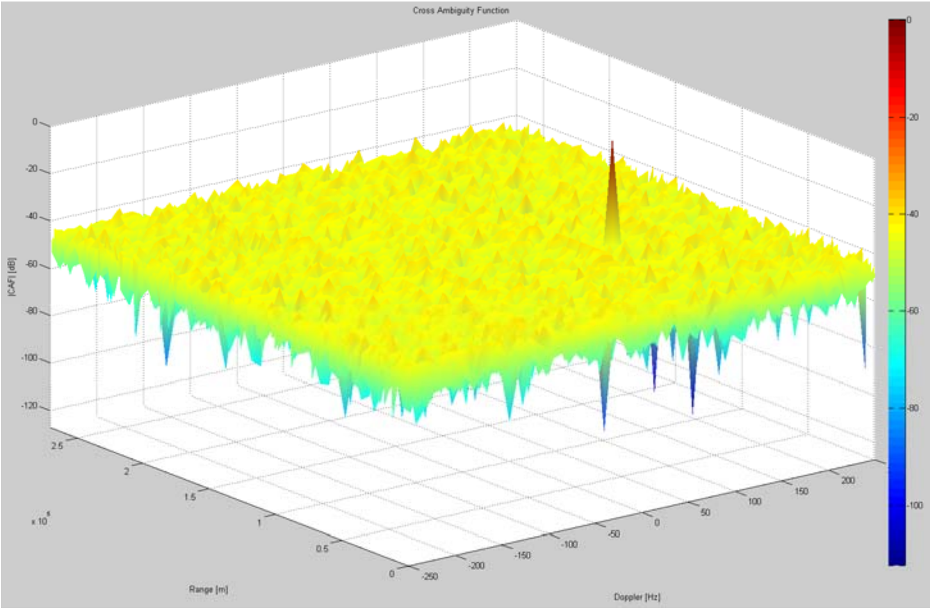

Fig.1 - Example of CAF

1.2 The cross-correlation method of CAF

This method (2) is based on cross-correlation which is defined by:

$$R_{s_1[n]s_2[n]}(\tau) = \sum_{n=0}^{N-1} s_1[n]s_2^*[n+\tau] \tag{7}$$

We have $s_2^*[n+\tau] = s_2^*\left[n+\tau\right]e^{-j2\pi f_d \frac{n}{N}}$ from equation (3) and on substituting this to equation (7)

$$CAF(\tau, f) = \sum_{n=0}^{N-1} s_1[n]\left\{ s_2\left[n+\tau\right]e^{j2\pi f\frac{[n+\tau]}{N}} e^{-j2\pi f\frac{\tau}{N}} \right\}^* \equiv xcorr\left(s_1\left[n\right], s_2^*\left[n\right]\right) \tag{8}$$

Equation (8) is final term for computing CAF using cross-correlation method. As per the correlation algorithm, each Doppler shift (f) is computed for the entire range of time delays $\tau \in \langle -N, N \rangle$. The advantage of this method is that we can compute any possible resolution in Doppler range. Total amount of mathematical operations [7] in defined Doppler range is defined by

$$P_{xcorr} = 3N_f N\left(2+5\log_2 N\right) \text{ [flops]} \tag{9}$$

1.3 The FFT method of CAF

This method is based on discrete Fourier transform (DFT), which is defined as

$$S(f) = \sum_{n=0}^{N-1} s\left[n\right]e^{-j2\pi f\frac{n}{N}} \equiv DFT\left[s\left[n\right]\right] \tag{10}$$

Substituting $s\left[n\right] = s_1\left[n\right]s_2^*\left[n+\tau_d\right]$ equation (3) we get

$$CAF(\tau, f) = DFT\left[s_1[n]s_2^*[n+\tau_d]\right] \quad (11)$$

The DFT summation is normally computed using the Fast Fourier Transform (FFT) algorithm. According to the algorithm of this method, compute each time delays τ corresponding to Doppler shifts on whole range, which is defined $f \in \langle -N/2; N/2 \rangle$. The main disadvantage of this method is that real Doppler shifts are only small subset of computed whole range of Doppler shift. The most of computations of Doppler shifts are in range which is unnecessary because those correspond to the speed of target which is not real. Total amount of mathematical operations [7] in defined Doppler shifts is given by

$$P_{FFT} = 10 N_\tau N \log_2 N \ \left[flops\right] \quad (12)$$

2. SYSTEM SETTINGS AND METHODOLOGY OF TESTING

The basic input data based on requirement of passive radar system:

- Length of signals $s_1[n]$, $s_2[n]$ is $T = 0{,}8\ s$
- Sampling frequency $f_s = 200\ kHz$
- Frequency resolution $dF = 1\ Hz$
- Range resolution $dR = 1{,}5\ km$
- Maximal Range $R_{\max} = 270\ km$
- Maximal Doppler shift $F_{\max} = \pm 250\ Hz$
- Range of target $R = 100\ km$
- Doppler shift of target $F = 100$ Hz

The computation of CAF will be tested on PC with selected parameters:

- CPU Intel Core i5 – 2400 (4 cores)
- RAM 12 GB
- OS Windows 7 64 bit SP1
- Mathworks MATLAB R2011b – 64bit

The base method which will be chosen for reference will be *summation method* because this method computes CAF according to definition of CAF. The reference time will be computed for 1 core (without any parallelization). The methods of CAF will be repeated 20 times. Tab. 1 presents mean values. A time marked with * is reference time for computing speed up of computation. The speed-up computation is done as follows:

$$SpeedUp = \frac{\text{Reference Time}}{\text{Measured Time}} \ [-] \quad (13)$$

Tab. 2 shows the number of mathematical computation based on input data.

3. RESULTS

The measurement of computational complexity is done using Matlab system. Tab. 1 shows results of computation time for each method. It can be seen at Tab. 2 that number of mathematical operations in not only the function of computation time. It is interesting to note that though the cross correlation method has less number of mathematical computations, still have worse results than summation method. This is due to the fact the cross correlation method used built in *xcorr* function which has very high computational demands because Matlab does not implement *xcorr* using FFT algorithms rather it implements it by definition.

Tab. 1 – Table of computation time for each method with speed-up factor

	The summation method I. (computation with 2 loops)			**The summation method II.** (computation with 1 loop + matrix)		
Number of cores CPU	1	2	4	1	2	4
The Computation time [s]	22,75*	13,78	11,82	11,42	11,06	10,31
The Speed up [-]	1,00	1,65	1,92	1,99	2,06	2,21
	The Cross Correlation method I. (computation with 2 loops)			**The Cross Correlation method II.** (computation with 1 loop + matrix)		
Number of cores CPU	1	2	4	1	2	4
The Computation time [s]	79,67	41,42	31,16	72,71	40,80	28,66
The Speed up [-]	0,29	0,55	0,73	0,31	0,56	0,79
	The FFT method (computation with 2 loops)					
Number of cores CPU	1	2	4			
The Computation time [s]	2,10	1,40	1,19			
The Speed up [-]	10,84	16,20	19,04			

The differences between summation method I. and II. are as follows: The first algorithm used two *for-end* loops and second algorithm is particularly vectorized and use only 1 loop, other mathematical operations are done my matrix operation for which is Matlab more optimized. Similar differences are between first and second algorithm of cross correlation method.

The most important tasks are about resolution of cross ambiguity function which are defined for time delay (or range) and Doppler shift. The resolution is defined like minimal distance which should be between targets for theirs recognition. On Fig.2a, b which shows slices of CAF at maximum (position of target), 2a is slice at maximal range and 2b is slice at maximal Doppler shift. Blue line at both cases describing width of main lobe at half of main lobe which is typical value for describing resolution of radar systems. The value at dB unit is computed by form $dB = 10\log_{10}\left(0,5\right) = -3\ dB$.

The difference in Doppler shift at -3 dB is 1,39 Hz. If two targets have lower difference then is 1,39 at Doppler frequency then radar system can`t recognize both target. The difference in range is equal 1764 m, so if two or more targets will be closer radar system can`t recognize targets. The reason for both cases is that main lobes for both targets will be so close each other, and then results will look like one big main lobe and not two lobes (it is valid for Doppler and range slice too).

Tab. 2 – Table of number of mathematical computation

	The summation method	**The cross correlation method**	**The FFT method**
The number of mathematical operations	$1,385856 \cdot 10^{11}$	$1.702265 \cdot 10^{10}$	$4.978861 \cdot 10^{9}$

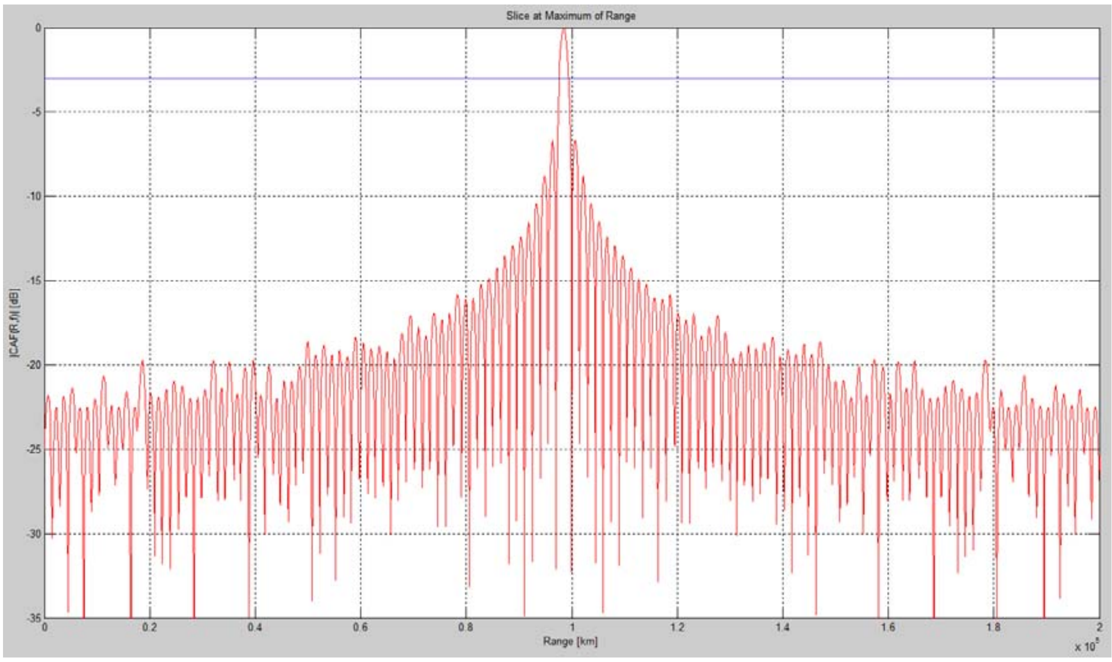

Fig. 2a - Slice at range

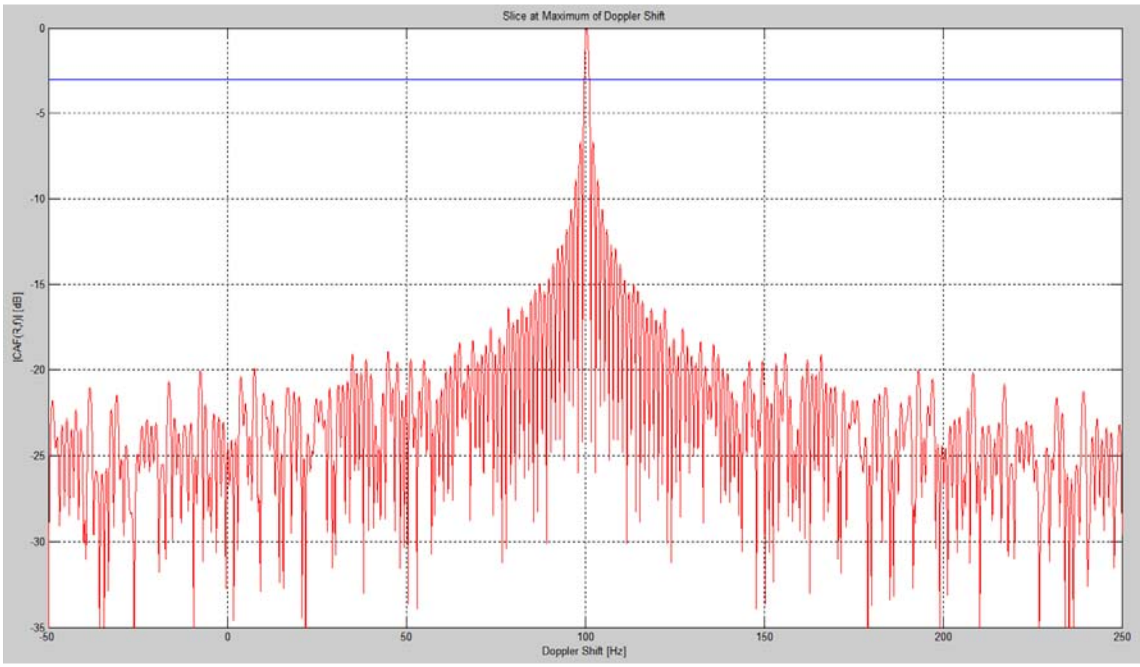

Fig. 2b - Slice at Doppler shift

CONCLUSION

This paper describes three “brute force” methods of computing cross ambiguity function. It has been demonstrated that numerical complexity is quite time-consuming and has huge resource requirements. The results of comparison of these methods show that the method using FFT is performs better than other methods due to high optimization of FFT. The speed-up of FFT methods on 4 cores CPU was almost 20x times in comparison to summation method which was chosen as reference method.

The CAF methods are compared in terms of the number of mathematical operations, and the resolution of CAF at Doppler shift and range is also computed.

The next step at optimization will be describing methods which use reduction of data. These methods will be compared with “brute force” method from point of view accuracy, resolution and time of computation. The primary requirement is the complete vectorization of codes, which brinks more speed up, because matrix operations are much faster than loops on the systems like Matlab.

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