NUMERICAL SIMULATION OF BODY MOVEMENTS

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Summary: This article discusses the problems of body motion in three-dimensional space and numerical simulations using Newton-Euler motion equations.

Key words: multibody, bodies, kinematics, dynamics, motion equations

INTRODUCTION

Numerical solutions of moving objects in three dimensional space are an important theme, especially for mechanism simulations. These simulations are solved with applications called multi-body systems, where mechanisms (Fig. 1) are assembled from rigid bodies, kinematic linkages, and other force interaction elements (e.g. external forces, springs, dampers, etc.). The method of solution described above is suitable for use in a computing environment of multibody systems.

Source: Authors

Fig. 1 - Example of typical multibody mechanism

For the purposes of this article, only the solution to determine the body movements if all acting forces are known is going to be discussed. This kind of body (Fig. 2) can be defined as absolutely rigid, with non-zero and positive weight and with main moments of inertia. These values are related to the body's center of gravity. Furthermore, the body may contain other general points whose position is, again, determined relatively to the position of the body's center of gravity. Each common point of the body is therefore represented by their own local

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coordinate system (LCS), while the position and orientation of the body itself, i.e. its center of gravity, will be determined in the global coordinate system (GCS).

Each of the points can be used, for example, as an acting place of external forces or moments, references for kinematic constraints, kinematic sensors, etc.

1.1 Calculation of body movement

The position of each object in GCS can be defined using the two data structures, where

$$\begin{bmatrix} cos\theta + e_x^2(1-cos\theta) & e_x e_y(1-cos\theta) - e_z sin\theta & e_x e_z(1-cos\theta) + e_y sin\theta \\ e_y e_x(1-cos\theta) + e_z sin\theta & cos\theta + e_y^2(1-cos\theta) & e_y e_z(1-cos\theta) - e_x sin\theta \\ e_z e_x(1-cos\theta) - e_y sin\theta & e_z e_y(1-cos\theta) + e_x sin\theta & cos\theta + e_z^2(1-cos\theta) \end{bmatrix} \quad (1)$$

the first is a vector specifying displacement of the body's center of gravity to the GCS origin, and the second is the transformation matrix for transformation from the local body space to the global space. For description of the rotation (i.e. orientation) the transformation matrix C for rotation around the general axis (Equation 1) is used (1).

Where e_x, e_y and e_z are the unit direction vector components of the general axis of rotation and θ is the angle of rotation.

The movement of body is then in each iteration determined by the Newton-Euler equations of motion. In the environment of the multibody system, the entire solution could be as follows:

- Determining of the force and torque results in the center of gravity.
- Updating the kinematic state of the center of the gravity.
- Updating the kinematic state of other common points of the body.

1.2 Determining of the force and torque results in the center of gravity

Points of the body (Fig. 2) are indexed from zero to N-1, where n is the number of points. The first point (i.e. point with zero index) is the body's center of gravity.

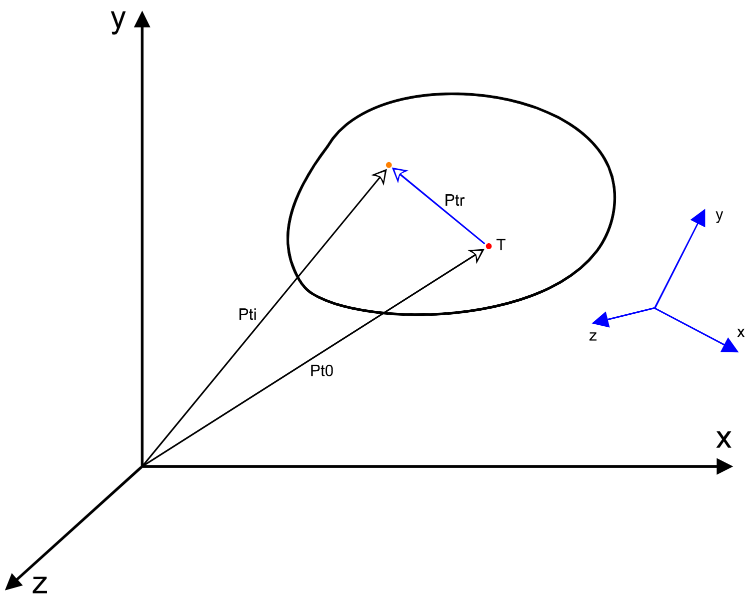

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Fig. 2 - Body with blue highlighted rotation of the local coordinate system

Force and torque resultants in the body's center of gravity (Equations 2, 3) are determined as a vector sum of the forces and moments in all of the points of the body. If gravitational acceleration is acting on the body, it can also be included in the resultant equations (Equation 2).

$$\vec{F}_{cg} = \sum_{i=0}^{n-1} \vec{F}_i + m \cdot \vec{g} \tag{2}$$

$$\vec{M}_{cg} = \sum_{i=0}^{n-1} (\vec{M}_i + \vec{P}_{tr} \times \vec{F}_i) \tag{3}$$

1.3 Updating the kinematic state of the center of the gravity

Body's kinematic state, or its center of the gravity, is determined by the Newton-Euler motion equations. Translational component arises from classical Newtonian motion equations (Equation 4), the velocity and position is obtained by subsequent integration (Equation 5, 6).

$$\vec{A}_t = \frac{\vec{F}_{cg}}{m} \tag{4}$$

$$\vec{V}_t = \int \vec{A}_t \cdot dt \tag{5}$$

$$\vec{P}_t = \int \vec{V}_t \cdot dt \tag{6}$$

Angular acceleration $\boldsymbol{A_r}$ is determined on the basis of Euler's motion equation (Equation 7), where it is necessary to transform torque vector $\boldsymbol{M_{cg}}$ and angular velocity vector $\boldsymbol{V_r}$ using transformation matrix $\boldsymbol{C}$ to the local coordinate system of the body (determined by orientation of the center of the gravity), and then the result is transformed back to the global coordinate system (2). It is because of the correct calculation of the inertia tensor which is, in contrast to the weight, directional. Angular velocity $\boldsymbol{V_r}$ is determined by simple integration of angular acceleration (Equation 8).

$$\vec{A}_r = C \cdot \left(I^{-1} \cdot \left(C' \cdot \vec{M}_{cg} - \left(C' \cdot \vec{V}_r \right) \times I \cdot \left(C' \cdot \vec{V}_r \right) \right) \right) \tag{7}$$

$$\vec{V}_r = \int \vec{A}_r \cdot dt \tag{8}$$

To obtain the orientation of the body, which is represented by the transformation matrix of rotation around the arbitrary axis (Equation 2), is not possible to use integration of the velocity vector in a three dimensional space, as with the translational movement (this is possible only in a plane) However, it is necessary to utilize a matrix process to compose rotations. In the form working with the discretization step $\boldsymbol{\Delta t}$, the calculation then has following form: first, the angular velocity vector $\boldsymbol{V_r}$ (Equation 8) is determined after the normalization of the direction of rotation axis $\boldsymbol{e}$ (Equation 9) and by using the discretization step it is possible to compute the appropriate angle of rotation $\boldsymbol{\theta}$ (Equation 10).

$$\vec{e} = \frac{\vec{V_r}}{\left| \vec{V_r} \right|} \tag{9}$$

$$\theta = \left| \vec{V_r} \right| \cdot \Delta t \tag{10}$$

Note: The program must be protected against a case where in equation (9) would a division by zero occur.

Subsequently, by using the transformation matrix (Equation 1) the incremental transformation matrix is calculated, and by multiplying it with matrix of rotational position from the previous discretization step $\boldsymbol{k_{-1}}$ the new rotational position for actual time is determined (Equation 11).

$$P_{r_k} = C(\vec{e}, \theta) \cdot P_{r_{k-1}} \tag{11}$$

1.4 Updating the kinematic state of other common points of the body

After the kinematic state of center of the gravity is calculated, all of other kinematic states in the remaining points of the body can be easily determined. The translational position of the point in GCS is determined by transformation and addition of relative displacement vector to the translational position of the center of gravity (Equation 12). Vectors of translational velocity and accelerations are calculated as the numerical derivation of positions, or acceleration (Equation 14, 16). Orientation (i.e. “rotational position”) is the product of a transformation matrix of the center of gravity and the transformation matrix in the relevant point (Equation 13). Angular acceleration and angular velocity of all points of the body are identical, so their values can be determined directly from point 0 (Equation 15, 17), which were determined when calculating the kinematic state of the body’s center of gravity of the body.

$$\forall i \in \left\langle 1 \middle| n-1 \right\rangle :$$

$$\vec{P}_{t_i} - \vec{P}_{t_0} + P_{r_0} \cdot \vec{P}_{tr_i} \tag{12}$$

$$P_{r_i} = P_{rr_i} \cdot P_{r_0} \tag{13}$$

$$\vec{V}_{t_i} = \frac{\Delta \vec{P}_{t_i}}{\Delta t} \tag{14}$$

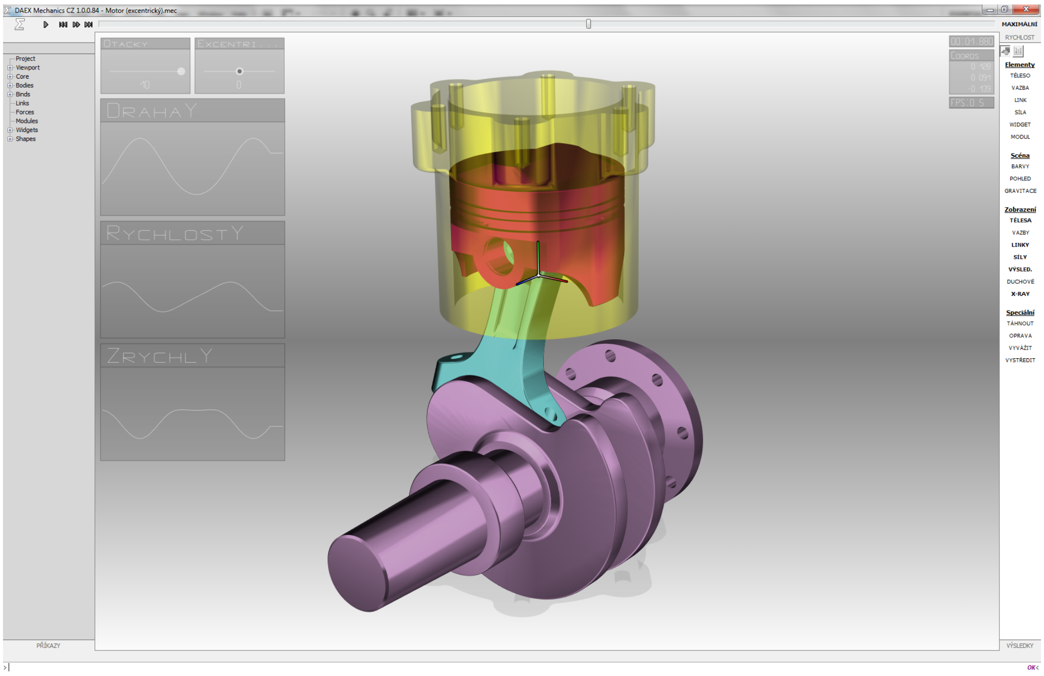
$$\vec{V}_{r_i} = \vec{V}_{r_0} \tag{15}$$

$$\vec{A}_{t_i} = \frac{\Delta \vec{V}_{t_i}}{\Delta t} \tag{16}$$

$$\vec{A}_{r_i} = \vec{A}_{r_0} \tag{17}$$

CONCLUSION

The method described in this article provides a simple and reliable approach to numerical solution of motion of bodies in three dimensional space (Fig. 3). Using the transformation matrix for rotation around the arbitrary axis, elegantly avoids the problems arising from the composition of rotations in space. The advantages are also easy and straightforward implementations using conventional programming languages, type C, Pascal, Matlab, etc.



Source: Authors

Fig. 3 - Multibody application based on described method

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